Analyse statique de programmes

CentraleSupélec – année 2024/2025

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CEA List
Laboratoire de Sûreté et Sécurité des Logiciels

Rappels

- Pour garantir qu'un programme est correct, il faut *vérifier formellement* des propriétés sur sa sémantique
- Il n'y a pas de procédure automatique qui permet de traiter tous les cas de manière exacte (Théorème de Rice)
- Nous avons défini la sémantique du langage While, et comment établir le *graphe de flot de contrôle*

- Les propriétés “utiles” peuvent se formaliser comme des éléments d'un treillis
- L'analyse *flot de données* se fait en résolvant une équation de point-fixe dans ce treillis à l'aide des itérations de Kleene
- En particulier, nous avons détaillé la sémantique collectrice et vu qu'elle n'est pas toujours calculable

Domaines abstraits

Principe de l'interprétation abstraite

- domaine concret : sémantique collectrice
- définir un domaine abstrait
- définir les fonctions de correspondance entre domaine concret et abstrait
  - abstraction α
  - concrétisation γ
- calcul dans le monde abstrait
  - environnements abstraits
  - évaluation abstraite des expression
  - fonction de transition abstraite
- analyse flot de données avec ces opérations abstraites
- correction de l'analyse : le point fixe abstrait capture-t-il toutes les traces concrètes possibles ?

Objectif

$$i^\flat \xrightarrow{F^\flat} \xrightarrow{F^\flat} \xrightarrow{F^\flat} \xrightarrow{F^\flat} \cdots\cdots\cdots \xrightarrow{{F^\flat}^{+\infty}} \mathrm{lfp}(F^\flat)$$

Objectif

$$\begin{tikzcd} i^\flat \arrow[r, "F^\flat"] \arrow[d, bend right, red, "\alpha"'] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & {} \arrow[r, dotted] & {} \arrow[r, "{F^\flat}^{+\infty}"] & \mathrm{lfp}(F^\flat) \\ i^\sharp \end{tikzcd}$$

Objectif

$$\begin{tikzcd}
i^\flat \arrow[r, "F^\flat"] \arrow[d, bend right, "\alpha"'] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & \cdots \arrow[r, "{F^\flat}^{+\infty}"] & \mathrm{lfp}(F^\flat) & \\
i^\sharp \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & \cdots \arrow[rr, "{F^\sharp}^{N}"'] & & \mathrm{lfp}(F^\sharp)
\end{tikzcd}$$

Objectif

$$
\begin{tikzcd}
i^\flat \arrow[r, "F^\flat"] \arrow[d, bend right, "\alpha"'] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & \cdots \arrow[r, "{F^\flat}^{+\infty}"] & \mathrm{lfp}(F^\flat) & \gamma(\mathrm{lfp}(F^\sharp)) \\
i^\sharp \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & \cdots \arrow[rr, "{F^\sharp}^N"'] & & \mathrm{lfp}(F^\sharp) \arrow[u, bend right, "\gamma"']
\end{tikzcd}
$$

OBJECTIF

$$
\begin{tikzcd}[column sep=small]
i^\flat \arrow[r, "F^\flat"] \arrow[d, bend right, "\alpha"'] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & {} \arrow[r, "F^\flat"] & {} \arrow[r, dotted, dash] & {} \arrow[r, "{F^\flat}^{+\infty}"] & \mathrm{lfp}(F^\flat) \sqsubseteq \gamma(\mathrm{lfp}(F^\sharp)) \\
i^\sharp \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & {} \arrow[r, "F^\sharp"'] & {} \arrow[r, dotted, dash] & {} \arrow[r, "{F^\sharp}^N"'] & \mathrm{lfp}(F^\sharp) \arrow[u, bend right, "\gamma"']
\end{tikzcd}
$$

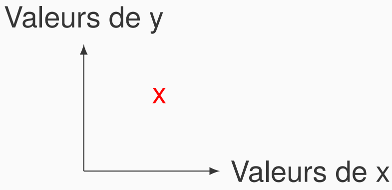
Une question de choix

En pratique, bien choisir son domaine abstrait est fondamental

- doit être suffisament précis
- en particulier, doit permettre d'énoncer la propriété souhaitée
- doit être calculable pour un coût temps/mémoire raisonnable (heures de calcul, Go de RAM pour des programmes réels)
- domaines non relationels : aucune relation conservée entre les éléments du domaine $\Rightarrow$ peu précis mais pas cher
- domaines relationnels : relations entre éléments du domaine $\Rightarrow$ plus précis mais plus cher

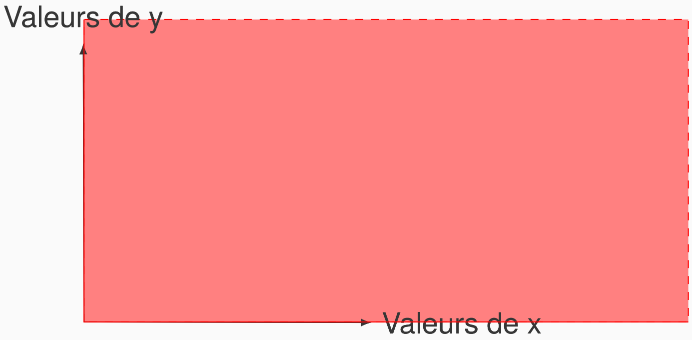
Domaines des constantes

- $x = z$ $(z \in \mathbb{Z})$
- domaine non relationnel
- si la valeur exacte n'est pas connue, perte de toute information



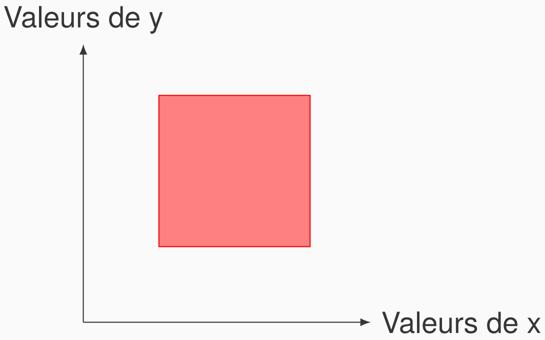
DOMAINE DES SIGNES

- x op 0 op $\in \{\geq, >, \leq, <, =, \neq\}$
- domaine non relationnel
- conservation de la polarité des valeurs possibles



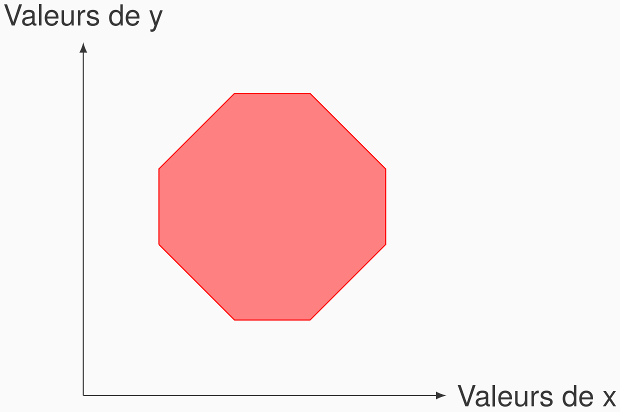
Domaine des intervalles

- $x \in [i_0, i_1]$
- domaine non relationnel
- conservation d'un intervalle regroupant toutes les valeurs possibles



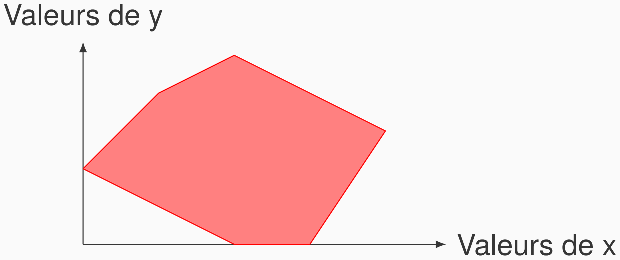
Domaine des octogones

- $\pm x \pm y \leq c$
- domaine relationnel
- conservation de relations linéaires simples entre élements



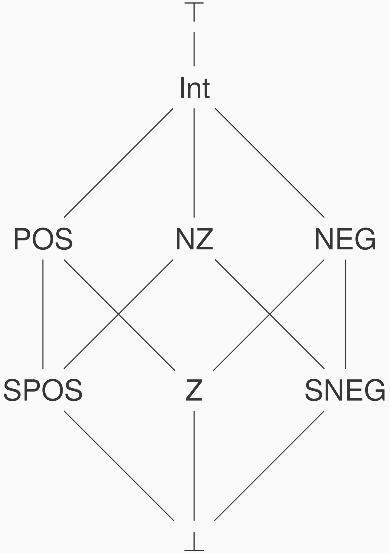
Domaine des polyèdres

- $kx + ly \leq c$
- domaine relationnel
- relations linéaires complexes entre éléments



Exemple : Treillis des signes

Définition



Environnements abstraits

On va considérer des environnements abstraits, $\mathbb{E}^\sharp : Var \rightharpoonup$ **Signes**. Chacun représente un ensemble d'environnements concrets :

$$\begin{aligned}
\gamma(\rho^\sharp) &= \{\rho^\flat \mid \forall x, \rho^\flat(x) \in \gamma(\rho^\sharp(x))\} \\
\alpha(\{\rho_i^\flat\}) &= x \mapsto \alpha(\{\rho_i^\flat(x)\})
\end{aligned}$$

Opérations abstraites

Les fonctions de transitions sont assez similaires aux opérations concrètes, sauf qu'on évalue les expressions dans **Signes**

$+^\sharp$	$\perp$	$\top$	Int	POS	NEG	NZ	SPOS	Z	SNEG
$\perp$									
$\top$									
Int									
POS									
NEG									
NZ									
SPOS									
Z									
SNEG									

Les fonctions de transitions sont assez similaires aux opérations concrètes, sauf qu'on évalue les expressions dans **Signes**

$+^\sharp$	$\bot$	$\top$	Int	POS	NEG	NZ	SPOS	Z	SNEG
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$
$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
Int	$\bot$	$\top$	Int	Int	Int	Int	Int	Int	Int
POS	$\bot$	$\top$	Int	POS	Int	Int	SPOS	POS	Int
NEG	$\bot$	$\top$	Int	Int	NEG	Int	Int	NEG	SNEG
NZ	$\bot$	$\top$	Int	Int	Int	Int	Int	NZ	Int
SPOS	$\bot$	$\top$	Int	SPOS	Int	Int	SPOS	SPOS	Int
Z	$\bot$	$\top$	Int	POS	NEG	NZ	SPOS	Z	SNEG
SNEG	$\bot$	$\top$	Int	Int	SNEG	Int	Int	SNEG	SNEG

Fonctions de transition abstraite

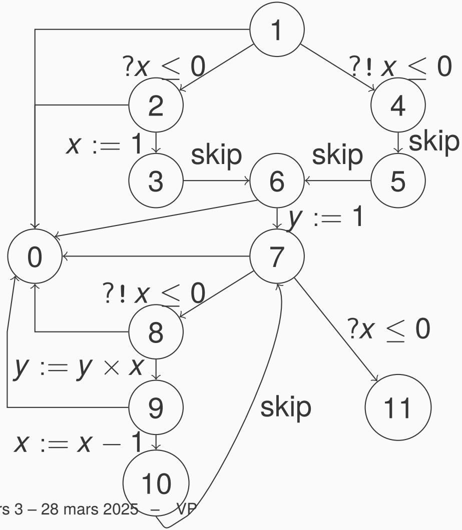
Skip
$$\frac{}{\rho^\sharp \to^\sharp_{\mathsf{skip}} \rho^\sharp}$$

Assignement
$$\frac{\rho^\sharp \vdash e \Downarrow^\sharp_\epsilon v \qquad v \notin \{\bot \mid \mathsf{Erreur}\}}{\rho^\sharp \to^\sharp_{x:=e} \rho^\sharp[x \mapsto v]}$$

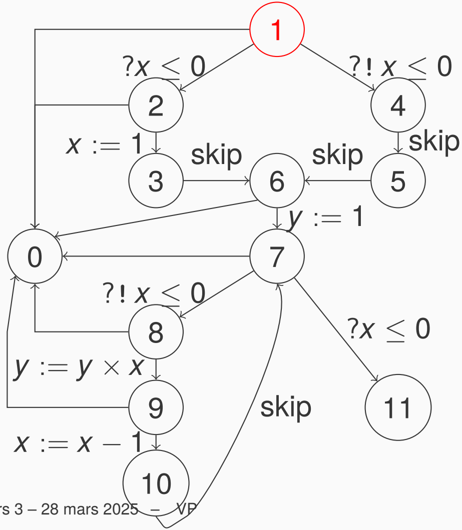
Cond
$$\frac{\rho^\sharp \vdash e \Downarrow^\sharp_\epsilon v \qquad v \notin \{\mathsf{Z} \mid \bot \mid \mathsf{Erreur}\}}{\rho^\sharp \to^\sharp_{?e} \lfloor \rho^\sharp \rfloor}$$

Err
$$\frac{\rho^\sharp \vdash e \Downarrow^\sharp_\epsilon v \qquad v \in \{\top \mid \mathsf{Erreur} \mid \bot\}}{\rho^\sharp \to^\sharp_{Err(e)} \rho^\sharp}$$

Analyse



nœud	x	y
s_1		
s_2		
s_3		
s_4		
s_5		
s_6		
s_7		
s_8		
s_9		
s_{10}		
s_{11}		
s_0		



nœud	x	y
s_1	Int	Int
s_2		
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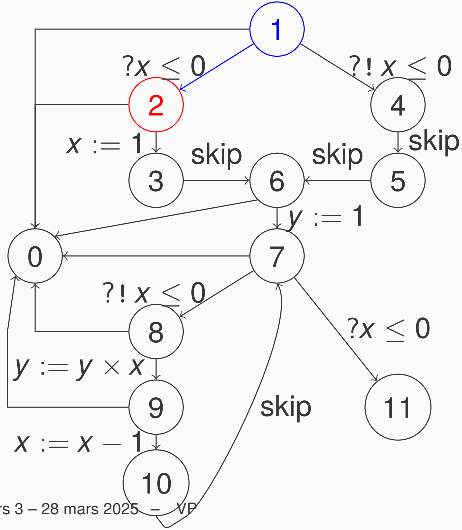
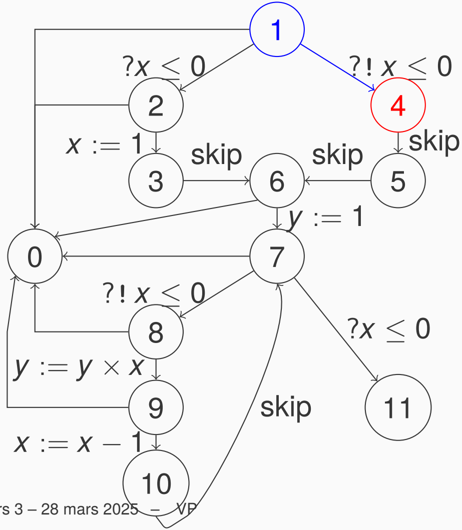
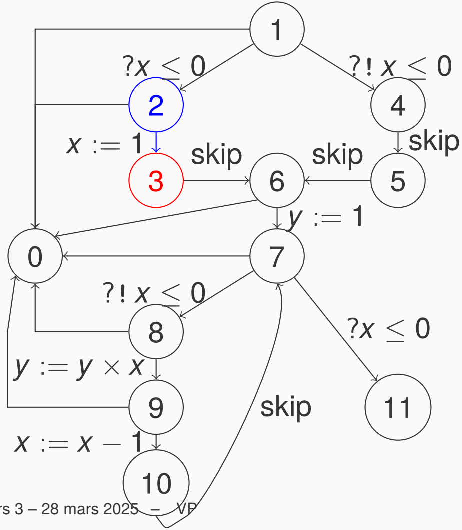
s_{11}		
s_0		



nœud	x	y
s_1	Int	Int
s_2	NEG	Int
s_3		
s_4		
s_5		
s_6		
s_7		
s_8		
s_9		
s_{10}		
s_{11}		
s_0		



nœud	x	y
s_1	Int	Int
s_2	NEG	Int
s_3		
s_4	SPOS	Int
s_5		
s_6		
s_7		
s_8		
s_9		
s_{10}		
s_{11}		
s_0		



nœud	x	y
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s_2	NEG	Int
s_3	SPOS	Int
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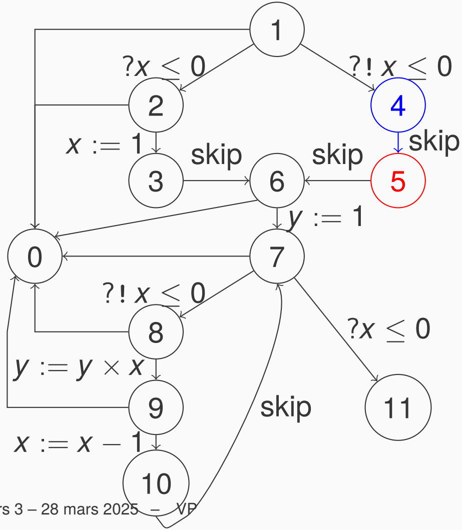
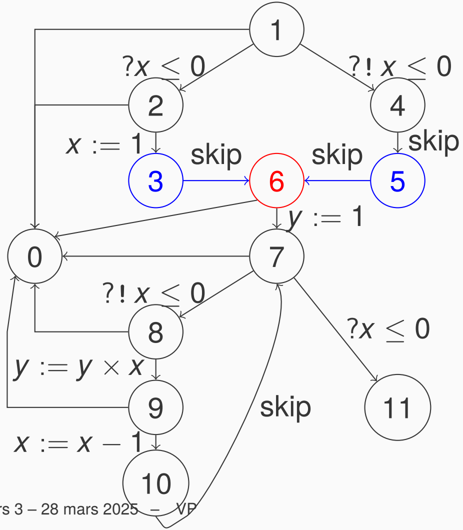
s_{11}		
s_0		



nœud	x	y
s_1	Int	Int
s_2	NEG	Int
s_3	SPOS	Int
s_4	SPOS	Int
s_5	SPOS	Int
s_6		
s_7		
s_8		
s_9		
s_{10}		
s_{11}		
s_0		



nœud	x	y
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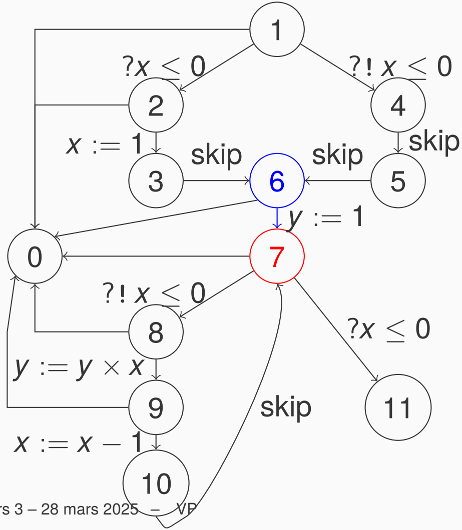
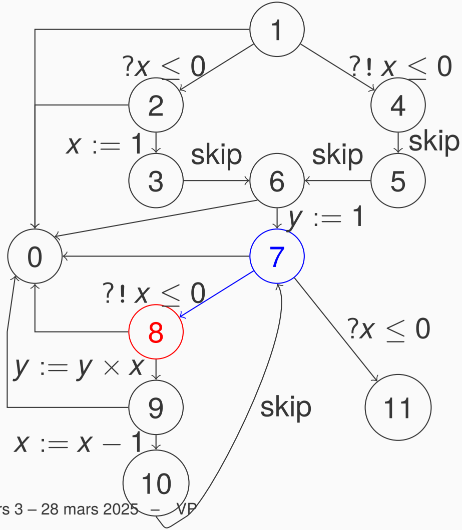
s_0		



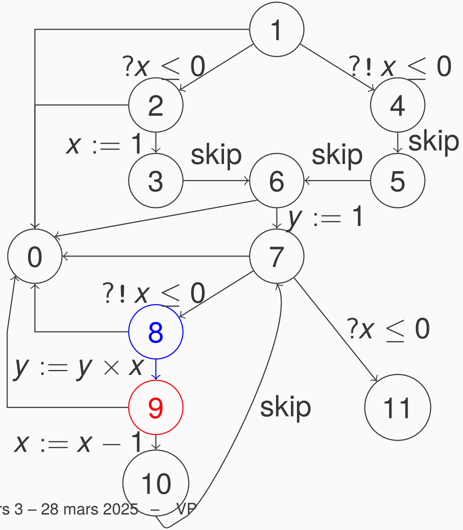
nœud	x	y
s_1	Int	Int
s_2	NEG	Int
s_3	SPOS	Int
s_4	SPOS	Int
s_5	SPOS	Int
s_6	SPOS	Int
s_7	SPOS	SPOS
s_8		
s_9		
s_{10}		
s_{11}		
s_0		

ANALYSE



nœud	x	y
s_1	Int	Int
s_2	NEG	Int
s_3	SPOS	Int
s_4	SPOS	Int
s_5	SPOS	Int
s_6	SPOS	Int
s_7	SPOS	SPOS
s_8	SPOS	SPOS
s_9		
s_{10}		
s_{11}		
s_0		

Analyse



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s_5	SPOS	Int
s_6	SPOS	Int
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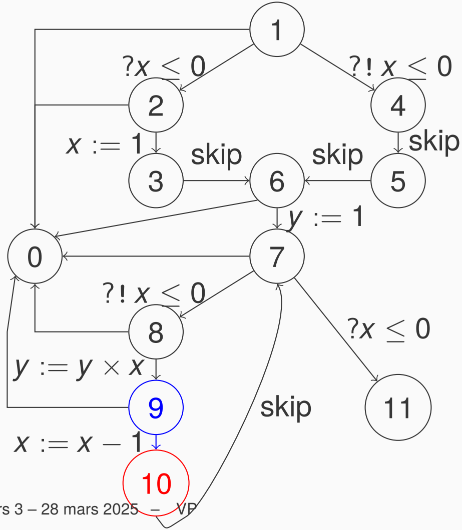
s_9	SPOS	SPOS
s_{10}		
s_{11}		
s_0		



nœud	x	y
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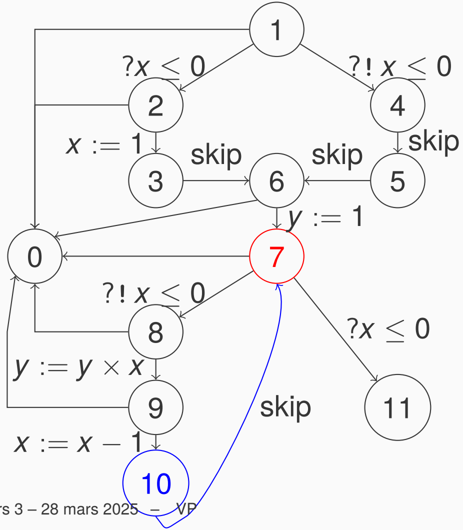
s_4	SPOS	Int
s_5	SPOS	Int
s_6	SPOS	Int
s_7	SPOS	SPOS
s_8	SPOS	SPOS
s_9	SPOS	SPOS
s_{10}	Int	SPOS
s_{11}		
s_0		



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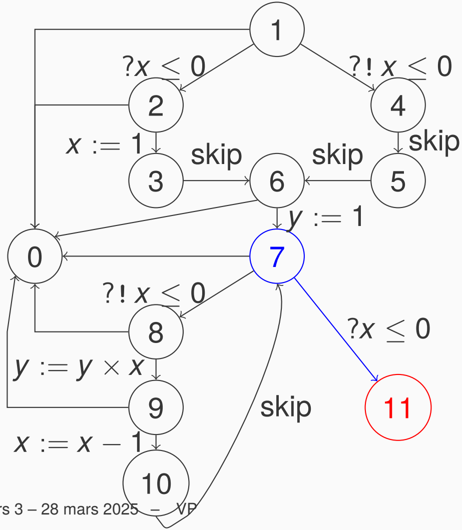
s_4	SPOS	Int
s_5	SPOS	Int
s_6	SPOS	Int
s_7	Int	SPOS
s_8	SPOS	SPOS
s_9	SPOS	SPOS
s_{10}	Int	SPOS
s_{11}		
s_0		



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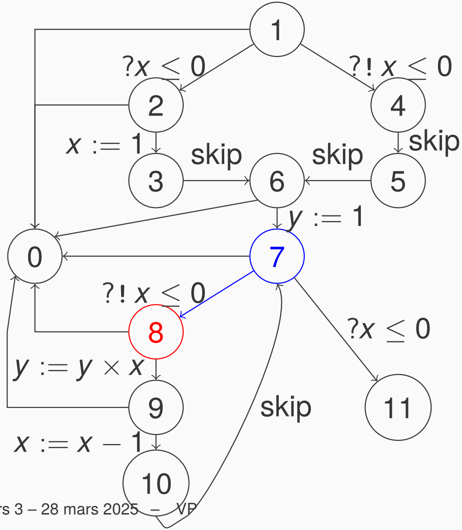
s_7	Int	SPOS
s_8	SPOS	SPOS
s_9	SPOS	SPOS
s_{10}	Int	SPOS
s_{11}	NEG	SPOS
s_0		



nœud	x	y
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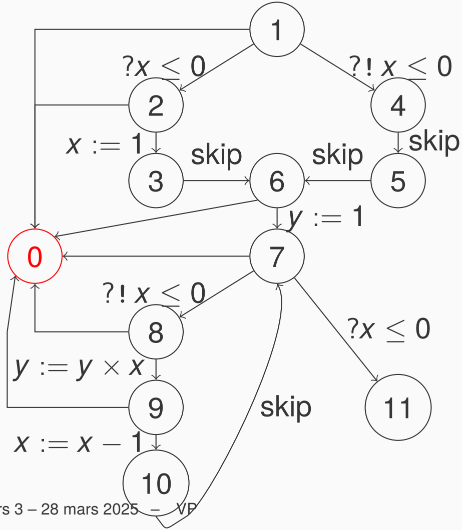
s_4	SPOS	Int
s_5	SPOS	Int
s_6	SPOS	Int
s_7	Int	SPOS
s_8	SPOS	SPOS
s_9	SPOS	SPOS
s_{10}	Int	SPOS
s_{11}	NEG	SPOS
s_0		



nœud	x	y
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s_2	NEG	Int
s_3	SPOS	Int
s_4	SPOS	Int
s_5	SPOS	Int
s_6	SPOS	Int
s_7	Int	SPOS
s_8	SPOS	SPOS
s_9	SPOS	SPOS
s_{10}	Int	SPOS
s_{11}	NEG	SPOS
s_0	$\perp$	$\perp$

Propriétés

- [Terminaison] L'analyse des signes termine toujours.
- [Correction] Soit s un sommet du graphe de contrôle de P et x une variable. $Coll_P(s)(x) \in \gamma(\rho^\sharp(s)(x))$ où $\rho^\sharp(s)$ est l'environnement associé à s par l'analyse.
- La correction peut se montrer "à la main", en raisonnant par induction sur P, mais il existe une manière systématique de construire l'analyse qui garantit que c'est correct.

Correspondance de Galois

Abstraction et concrétisation

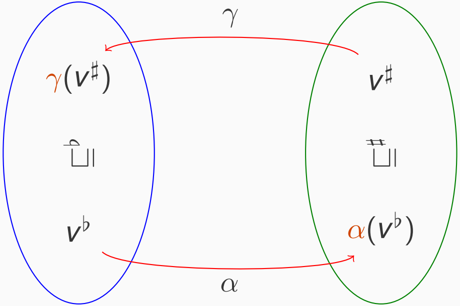
- treillis concret $(L^\flat, \sqsubseteq^\flat)$ pour l'analyse concrète
- treillis abstrait $(L^\sharp, \sqsubseteq^\sharp)$ pour l'analyse abstraite
- relation d'ordre $\sqsubseteq^\sharp$: "être moins précis que"
- opérateur d'abstraction α : transforme un environnement concret en un abstrait
- opérateur de concrétisation γ : transforme un environnement abstrait en un concret
- $$\begin{tikzcd} (L^\flat, \sqsubseteq^\flat) \arrow[r, bend left, "\alpha"] & (L^\sharp, \sqsubseteq^\sharp) \arrow[l, bend left, "\gamma"] \end{tikzcd}$$

Connexion de Galois

Définition

Soient $(L^\sharp, \sqsubseteq^\sharp)$ et $(L^\flat, \sqsubseteq^\flat)$, deux treillis complets.
Deux fonctions $\alpha : L^\flat \rightarrow L^\sharp$ et $\gamma : L^\sharp \rightarrow L^\flat$ forment une connexion de Galois si et seulement si :

$$\forall v^\sharp \in L^\sharp, \forall v^\flat \in L^\flat,$$
$$\alpha(v^\flat) \sqsubseteq^\sharp v^\sharp \iff v^\flat \sqsubseteq^\flat \gamma(v^\sharp)$$



Composition de α et γ

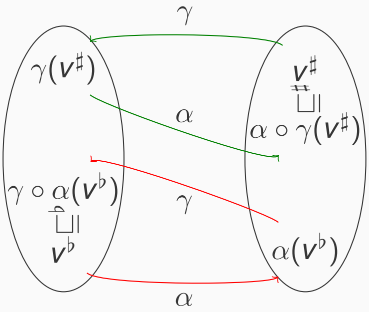
Lemme

(α, γ) est une connexion de Galois si et seulement si :

1 ⟩ α et γ sont monotones.

2 ⟩ $\forall v^\flat \in L^\flat,\ v^\flat \sqsubseteq^\flat (\gamma \circ \alpha)(v^\flat)$.

3 ⟩ $\forall v^\sharp \in L^\sharp,\ (\alpha \circ \gamma)(v^\sharp) \sqsubseteq^\sharp v^\sharp$



Insertion de Galois

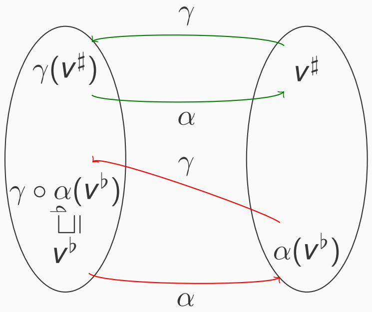
Définition

(α, γ) est une insertion de Galois si et seulement si les trois conditions suivantes sont satisfaites :

1 ⟩ α et γ sont monotones

2 ⟩ $\forall v^\flat \in L^\flat,\ v^\flat \sqsubseteq^\flat (\gamma \circ \alpha)(v^\flat)$

3 ⟩ $\forall v^\sharp \in L^\sharp,\ (\alpha \circ \gamma)(v^\sharp) = v^\sharp$



Connexion de Galois et analyse statique

Abstraction de la sémantique collectrice

- sémantique collectrice =
  point fixe d'une fonction $F^\flat$ dans $\mathcal{S} \to \mathcal{P}(Env^\flat)$
- domaine concret $L^\flat = \mathcal{S} \to \mathcal{P}(Env^\flat)$
- relation d'ordre concrète $f1 \sqsubseteq^\flat f2 \iff \forall s \in \mathcal{S}, f1(s) \subseteq f2(s)$
- but : trouver un domaine abstrait et une correspondance de Galois avec $(L^\flat, \sqsubseteq^\flat)$
- questions :
  - lien entre points fixes et connexions de Galois ?
  - lien entre espaces fonctionnels et connexions de Galois ?

Théorème (Cousot)

Soit une connexion de Galois entre treillis complet $L^\flat$ et $L^\sharp$:

$$\begin{tikzcd} L^\flat \arrow[r, bend left, "\alpha"] & L^\sharp \arrow[l, bend left, "\gamma"] \end{tikzcd}$$

Si $f : L^\flat \to L^\flat$ est une fonction monotone alors :

$$\mathrm{lfp}(f) \sqsubseteq^\flat \gamma(\mathrm{lfp}(\alpha \circ f \circ \gamma)).$$

Monotonie

Soit $(L, \sqsubseteq, \sqcup, \sqcap)$ un treillis complet, et f et g deux fonctions monotones de D dans D.

$$\text{si pour tout } x \in L, f(x) \sqsubseteq g(x), \ \text{alors } \mathrm{lfp}(f) \sqsubseteq \mathrm{lfp}(g).$$

Point fixe abstrait

$$
\begin{tikzcd}
\emptyset \arrow[rr, "{F^\flat}^{+\infty}"] \arrow[dd, "\alpha"'] & & \rho^\flat \arrow[d, phantom, "\rotatebox{-90}{$\subseteq$}"] \\
& & \gamma \circ \rho^\sharp \\
\bot \arrow[rr, "{F^\sharp}^{N}"'] & & \rho^\sharp \arrow[u, "\gamma"']
\end{tikzcd}
$$

Le théorème de Cousot nous donne un bon candidat pour $F^\sharp$:

$$F^\sharp = \alpha \circ F^\flat \circ \gamma$$

- correct “automatiquement” par construction
- mais pas forcément calculable ($F^\flat$ elle-même ne l’est pas)
- approximation plus grossière possible grâce au théorème de monotonie

Espace fonctionnel

Lemme

Si (α, γ) forme une connexion de Galois entre $(L^\flat, \sqsubseteq^\flat)$ et $(L^\sharp, \sqsubseteq^\sharp)$, et E est un ensemble, les fonctions (α', γ') définies par

- $\alpha'(f^\flat) = \lambda e.\alpha(f^\flat(e))$
- $\gamma'(f^\sharp) = \lambda e.\gamma(f^\sharp(e))$

forment une connexion de Galois entre $(E \to L^\flat)$ et $(E \to L^\sharp)$

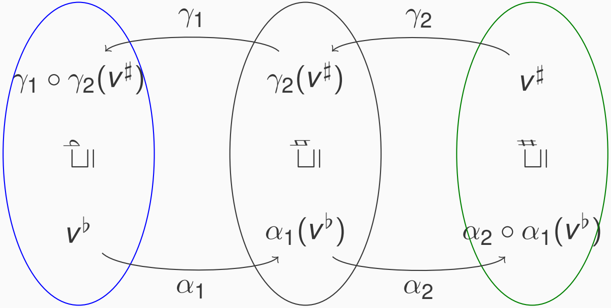
Ici, $L^\flat$ sont des ensembles d'environnements $\mathcal{P}(Env^\flat)$:

- rappel : $Env^\flat = Var \rightharpoonup \mathbb{Z}$
- trouver un domaine abstrait $Env^\sharp$
- trouver une connexion de Galois entre $\mathcal{P}(Env^\flat)$ et $Env^\sharp$
- utilisation d'un treillis intermédiaire $Env^{NR} = Var \rightharpoonup \mathcal{P}(\mathbb{Z})$, treillis des abstractions non-relationnelles.

Composition d'abstraction

Lemme

Soient $(L^\flat, \sqsubseteq^\flat)$ et $(L^\natural, \sqsubseteq^\natural)$ munis d'une connexion de Galois (α_1, γ_1) et $(L^\natural, \sqsubseteq^\natural)$, $(L^\sharp, \sqsubseteq^\sharp)$ munis d'une connexion de Galois (α_2, γ_2). Alors, $(\alpha_2 \circ \alpha_1, \gamma_1 \circ \gamma_2)$ forme une connexion de Galois entre $(L^\flat, \sqsubseteq^\flat)$ et $(L^\sharp, \sqsubseteq^\sharp)$. De même, si (α_1, γ_1) et (α_2, γ_2) sont des insertions de Galois, leur composition l'est aussi.



Abstraction non relationnelle

Lemme

$(\mathcal{P}(Env^\flat), \subseteq)$ et $(Var \rightharpoonup \mathcal{P}(\mathbb{Z}), \subseteq)$ peut être munie d'une connexion de Galois $(\gamma^{NR}, \alpha^{NR})$, définie de la manière suivante :

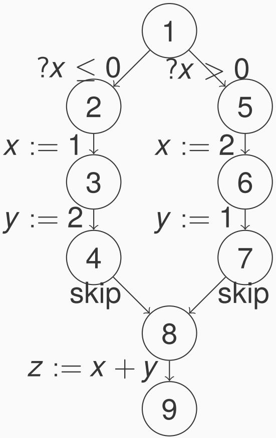
- $\alpha^{NR}(R^\flat) = \lambda x.\{\rho^\flat(x) \mid \rho^\flat \in R^\flat\}$
- $\gamma^{NR}(\rho^\sharp) = \{\rho^\flat \mid \forall x, \rho^\flat(x) \in \rho^\sharp(x)\}$

Remarque

aucune relation entre variables n'est préservée

Exemple non-relationnel

```
if x<=0 then x:=1; y:=2 else x:=2; y:=1 fi; z:=x+y
```



s_1

s_2

s_3

s_4

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s_9

Exemple non-relationnel

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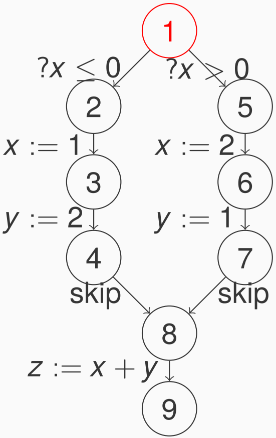

s_1

s_2

s_3

s_4

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Exemple non-relationnel

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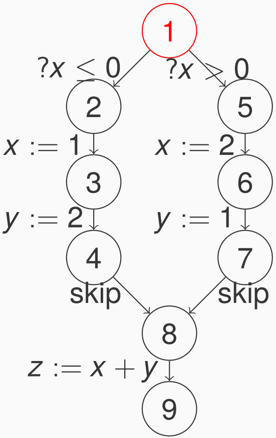

s_1 $\{[x \mapsto v]\}$
s_2
s_3
s_4
s_5
s_6
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s_8
s_9

Exemple non-relationnel

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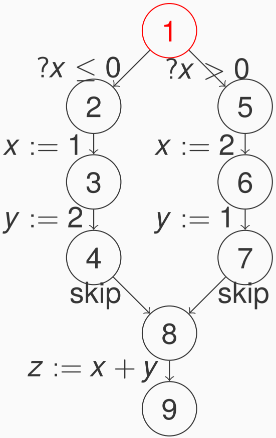

s_1 $\{[x \mapsto v]\}$ $[x \mapsto \mathbb{Z}]$

s_2

s_3

s_4

s_5

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Exemple non-relationnel

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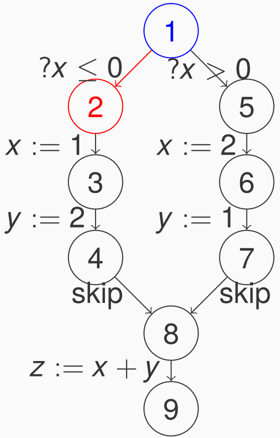

s_1 $\{[x \mapsto v]\}$ $[x \mapsto \mathbb{Z}]$

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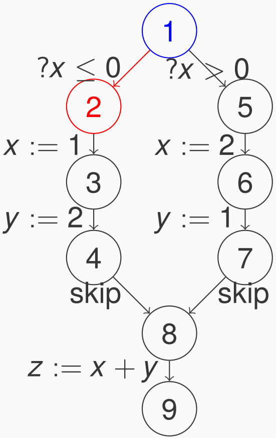

s_1 $\{[x \mapsto v]\}$ $[x \mapsto \mathbb{Z}]$

s_2 $\{[x \mapsto v]\}$

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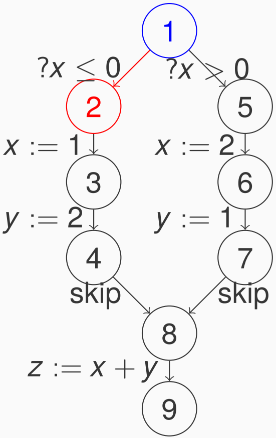

s_1	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}]$
s_2	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}^-]$
s_3		
s_4		
s_5		
s_6		
s_7		
s_8		
s_9		

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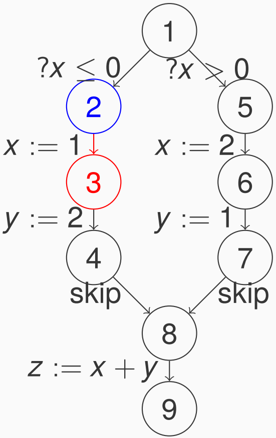

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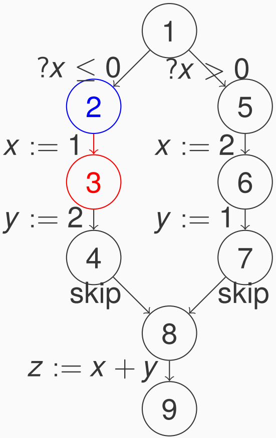

s_1 $\{[x \mapsto v]\}$ $[x \mapsto \mathbb{Z}]$

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s_4

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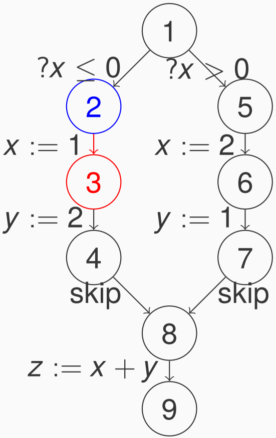

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s_4		
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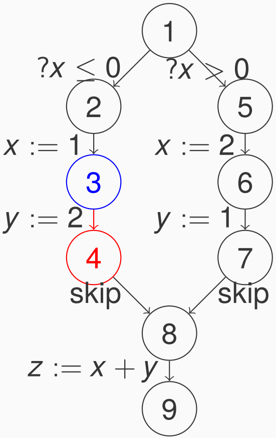

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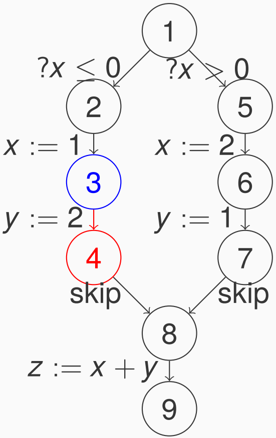

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s_4	$\{[x \mapsto 1, y \mapsto 2]\}$	
s_5		
s_6		
s_7		
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s_9		

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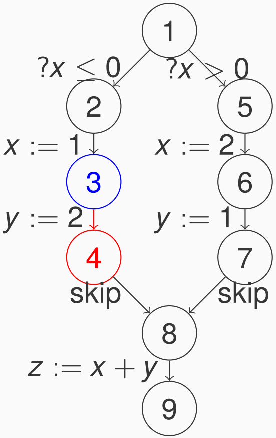

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s_5		
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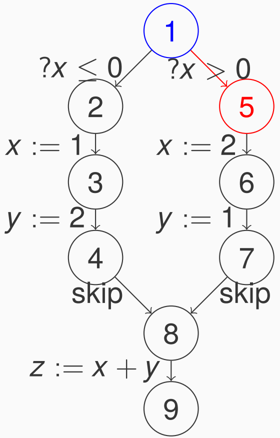

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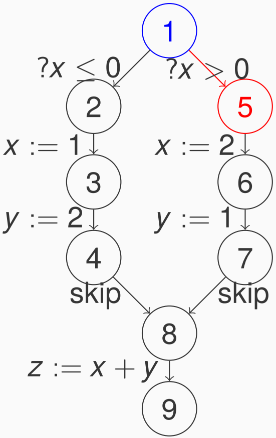

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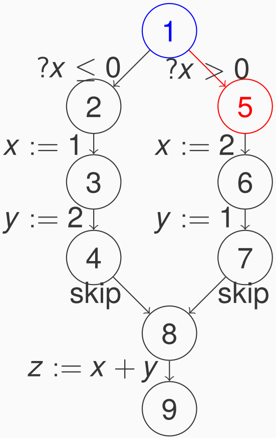

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EXEMPLE NON-RELATIONNEL

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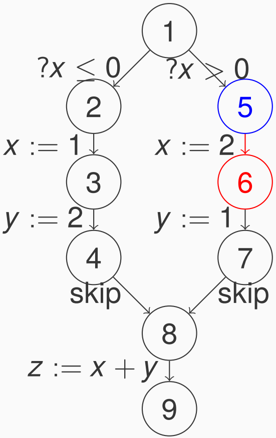

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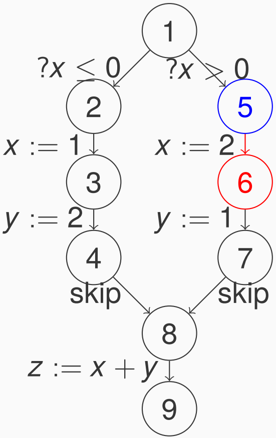

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s_7		
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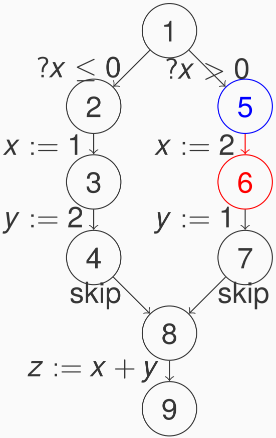

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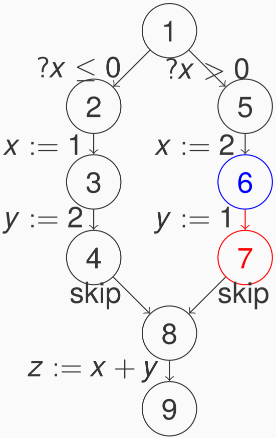

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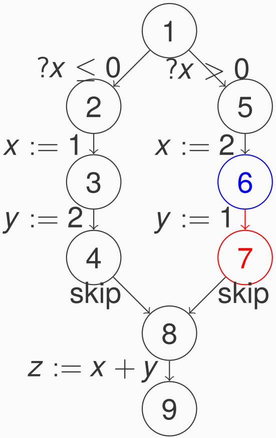

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s_3	$\{[x \mapsto 1]\}$	$[x \mapsto \{1\}]$
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s_5	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}^{+\star}]$
s_6	$\{[x \mapsto 2]\}$	$[x \mapsto \{2\}]$
s_7	$\{[x \mapsto 2, y \mapsto 1]\}$	
s_8		
s_9		

Exemple non-relationnel

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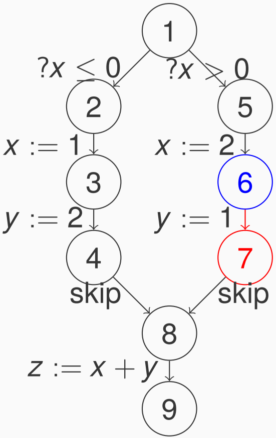
if x<=0 then x:=1; y:=2 else x:=2; y:=1 fi; z:=x+y
```



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s_8		
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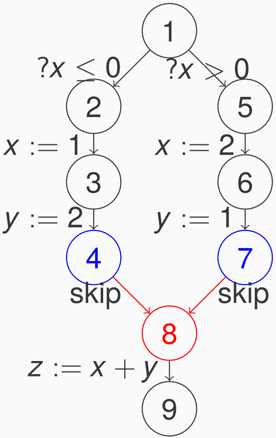

s_1	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}]$
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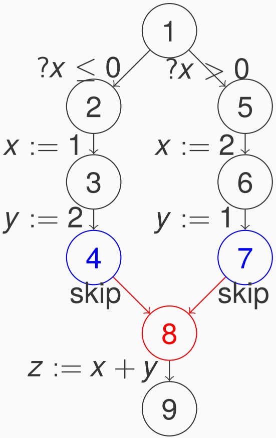

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s_8	$\left\{\begin{array}{c}[x \mapsto 1, y \mapsto 2]\\ [x \mapsto 2, y \mapsto 1]\end{array}\right\}$	
s_9		

Exemple non-relationnel

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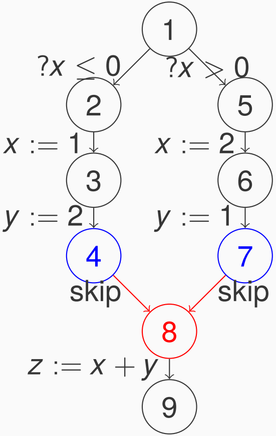

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s_8	$\left\{\begin{array}{l}[x \mapsto 1, y \mapsto 2] \\ [x \mapsto 2, y \mapsto 1]\end{array}\right\}$	$\left[\begin{array}{l}x \mapsto \{1,2\} \\ y \mapsto \{1,2\}\end{array}\right]$
s_9		

Exemple non-relationnel

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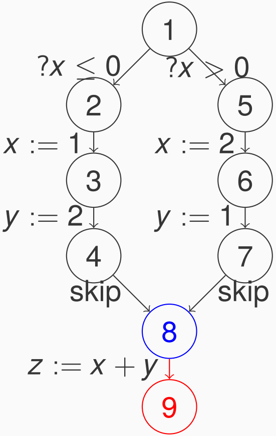

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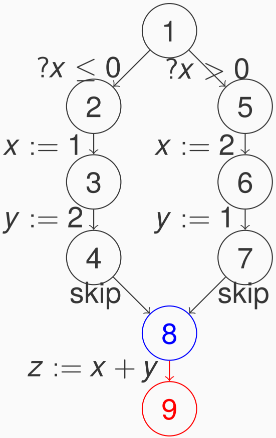
s_5	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}^{+\star}]$
s_6	$\{[x \mapsto 2]\}$	$[x \mapsto \{2\}]$
s_7	$\{[x \mapsto 2, y \mapsto 1]\}$	$[x \mapsto \{2\}, y \mapsto \{1\}]$
s_8	$\left\{\begin{array}{l}[x \mapsto 1, y \mapsto 2]\\ [x \mapsto 2, y \mapsto 1]\end{array}\right\}$	$\left[\begin{array}{l}x \mapsto \{1,2\}\\ y \mapsto \{1,2\}\end{array}\right]$
s_9		

EXEMPLE NON-RELATIONNEL

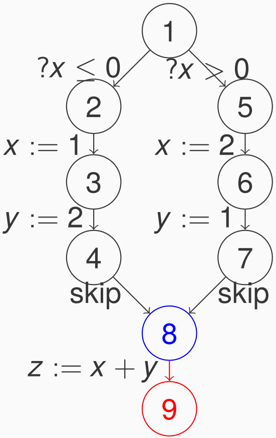
```
if x<=0 then x:=1; y:=2 else x:=2; y:=1 fi; z:=x+y
```



s_1	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}]$
s_2	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}^-]$
s_3	$\{[x \mapsto 1]\}$	$[x \mapsto \{1\}]$
s_4	$\{[x \mapsto 1, y \mapsto 2]\}$	$[x \mapsto \{1\}, y \mapsto \{2\}]$
s_5	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}^{+\star}]$
s_6	$\{[x \mapsto 2]\}$	$[x \mapsto \{2\}]$
s_7	$\{[x \mapsto 2, y \mapsto 1]\}$	$[x \mapsto \{2\}, y \mapsto \{1\}]$
s_8	$\left\{\begin{array}{c}[x \mapsto 1, y \mapsto 2] \\ [x \mapsto 2, y \mapsto 1]\end{array}\right\}$	$\left[\begin{array}{l}x \mapsto \{1,2\} \\ y \mapsto \{1,2\}\end{array}\right]$
s_9	$\left\{\begin{array}{c}\left[\begin{array}{l}x \mapsto 1, y \mapsto 2 \\ z \mapsto 3\end{array}\right] \\ \left[\begin{array}{l}x \mapsto 2, y \mapsto 1 \\ z \mapsto 3\end{array}\right]\end{array}\right\}$	

Exemple non-relationnel

```
if x<=0 then x:=1; y:=2 else x:=2; y:=1 fi; z:=x+y
```



s_1	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}]$
s_2	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}^-]$
s_3	$\{[x \mapsto 1]\}$	$[x \mapsto \{1\}]$
s_4	$\{[x \mapsto 1, y \mapsto 2]\}$	$[x \mapsto \{1\}, y \mapsto \{2\}]$
s_5	$\{[x \mapsto v]\}$	$[x \mapsto \mathbb{Z}^{+\star}]$
s_6	$\{[x \mapsto 2]\}$	$[x \mapsto \{2\}]$
s_7	$\{[x \mapsto 2, y \mapsto 1]\}$	$[x \mapsto \{2\}, y \mapsto \{1\}]$
s_8	$\left\{\begin{array}{l}[x \mapsto 1, y \mapsto 2] \\ [x \mapsto 2, y \mapsto 1]\end{array}\right\}$	$\left[\begin{array}{l}x \mapsto \{1,2\} \\ y \mapsto \{1,2\}\end{array}\right]$
s_9	$\left\{\begin{array}{l}\left[\begin{array}{l}x \mapsto 1, y \mapsto 2 \\ z \mapsto 3\end{array}\right] \\ \left[\begin{array}{l}x \mapsto 2, y \mapsto 1 \\ z \mapsto 3\end{array}\right]\end{array}\right\}$	$\left[\begin{array}{l}x \mapsto \{1,2\} \\ y \mapsto \{1,2\} \\ z \mapsto \{2,3,4\}\end{array}\right]$

Où en est-on ?

- souhait : un domaine abstrait pour $\mathcal{S} \to \mathcal{P}(\mathit{Env}^\flat)$
- réduction à un domaine abstrait pour $\mathcal{P}(\mathit{Env}^\flat)$ grâce au théorème de Cousot et au lemme des espace fonctionnels
- on va maintenant utiliser le domaine des signes pour abstraire les entiers
- $$(\mathcal{P}(\mathbb{Z}) \cup \{\texttt{erreur}\}, \subseteq) \underset{\gamma}{\overset{\alpha}{\leftrightarrows}} (\textbf{Signes}, \sqsubseteq)$$
- $$(\mathcal{P}(\mathit{Env}^\flat), \subseteq) \underset{\gamma^{NR}}{\overset{\alpha^{NR}}{\leftrightarrows}} (\mathit{Var} \rightharpoonup \mathcal{P}(\mathbb{Z}), \subseteq) \underset{\gamma^{\lambda}}{\overset{\alpha^{\lambda}}{\leftrightarrows}} (\mathit{Var} \rightharpoonup \textbf{Signes} \setminus \{\text{Erreur}\}, \subseteq)$$
- lemme des espaces fonctionnels pour définir $(\alpha^\lambda, \gamma^\lambda)$ à partir de $(\alpha^-, \gamma^-) \approx (\alpha, \gamma)$

Fonction de transition abstraite

Où va t-on ?

- la base : treillis des signes et définition de (α, γ), qui forme une insertion de Galois
- en corollaire immédiat : interprétation des environnements
- il n'y a ensuite plus qu'à calculer :
  - interprétation des opérateurs
  - interprétation des expressions
  - interprétation de la fonction de transition

Interprétation des opérateurs

Soit $L^\flat$ l'ensemble d'interprétation des éléments du langage (ici $\mathbb{Z}$) et (α, γ) une connexion de Galois entre $\mathcal{P}(L^\flat) \cup \{\texttt{erreur}\}$ et $L^\sharp$.
A tout opérateur concret $op^\flat$ (`+`, `*`, `=`, ...), on associe un opérateur abstrait $op^\sharp$:

$$op^\sharp(v_1^\sharp, v_2^\sharp, \cdots, v_n^\sharp) = \alpha(\{op^\flat(v_1^\flat, v_2^\flat, \cdots, v_n^\flat) \mid \forall i \in [1..n], v_i \in \gamma(v_i^\sharp)\})$$

Cette construction est "automatiquement" correcte :

Lemme

$\forall v_1, \ldots, v_n \in \mathbb{V}$, $\forall v_1^\sharp, \ldots, v_n^\sharp \in L^\sharp$,
si $v_i \in \gamma(v_i^\sharp)$ (pour $1 \leq i \leq n$),
alors $op(v_1, \cdots, v_n) \in \gamma(op^\sharp(v_1^\sharp, v_2^\sharp, \cdots, v_n^\sharp))$.

Remarque : on retrouve le $+^\sharp$ précédent. On peut calculer de même les autres opérateurs.

Interprétation des expressions

De la même manière, on définit une opération abstraite d'évaluation des expressions. On note $(\alpha^{Env}, \gamma^{Env}) = (\alpha^{\lambda} \circ \alpha^{NR}, \gamma^{NR} \circ \alpha^{\lambda})$.

$$E^{\sharp}(e, \rho^{\sharp}) = \alpha\left(\left\{ v^{\flat} \;\middle|\; \begin{array}{l} \exists \rho^{\flat} \in \gamma^{Env}(\rho^{\sharp}), \\ \quad \left[\begin{array}{l} \rho^{\flat} \vdash e \Downarrow_{\epsilon} v^{\flat} \text{ ; ou} \\ \forall v, \neg \rho^{\flat} \vdash e \Downarrow_{\epsilon} v \text{ et } v^{\flat} = \texttt{erreur} \end{array} \right. \end{array} \right\}\right)$$

Application : cas des variables

$$\frac{\mathsf{x} \in \mathrm{dom}(\rho) \qquad \rho(\mathsf{x}) = z}{\rho \vdash \mathsf{x} \Downarrow_\epsilon z}$$

$$\begin{aligned}
E^\sharp(\mathsf{x}, \rho^\sharp) &= \alpha\left(\left\{ v^\flat \;\middle|\; \begin{array}{l} \exists \rho^\flat \in \gamma^{Env}(\rho^\sharp), \\ \quad \left[\begin{array}{l} \rho^\flat \vdash \mathsf{x} \Downarrow_\epsilon v^\flat \text{ ; ou} \\ \forall v, \neg\rho^\flat \vdash \mathsf{x} \Downarrow_\epsilon v \text{ et } v^\flat = \mathtt{erreur} \end{array}\right. \end{array} \right\}\right) \\
&= \alpha\left(\left\{ v^\flat \;\middle|\; \begin{array}{l} \exists \rho^\flat \in \gamma^{Env}(\rho^\sharp), \\ \quad \left[\begin{array}{l} \rho^\flat(\mathsf{x}) = v^\flat \text{ ; ou} \\ \mathsf{x} \notin \mathrm{dom}(\rho^\flat) \text{ et } v^\flat = \mathtt{erreur} \end{array}\right. \end{array} \right\}\right) \\
&= \alpha\left(\left\{ v^\flat \;\middle|\; \begin{array}{l} v^\flat \in \gamma(\rho^\sharp(\mathsf{x})) \text{ ; ou} \\ \mathsf{x} \notin \mathrm{dom}(\gamma(\rho^\sharp)) \text{ et } v^\flat = \mathtt{erreur} \end{array} \right\}\right) \\
&= \begin{cases} \rho^\sharp(\mathsf{x}) & \text{si } \mathsf{x} \in \mathrm{dom}(\gamma(\rho^\sharp)) \\ \perp & \text{sinon} \end{cases}
\end{aligned}$$

Fonctions de transition abstraites

Rappel

Les fonctions de transition sont de la forme :

$$\mathcal{P}_e^\flat = \lambda R.\{\rho \mid \exists \rho' \in R, \rho' \to^\flat{}_e \rho\}$$

On cherche donc à calculer

$$\mathcal{P}_e^\sharp = \lambda \rho^\sharp.\alpha^{Env}(\{\rho \mid \exists \rho' \in \gamma^{Env}(\rho^\sharp), \rho' \to^\flat{}_e \rho\})$$

pour chacune des étiquettes skip, $x := e$, $?e$ et $Err(e)$.

Comme cette fonction n'est pas toujours calculable, on choisira $\mathcal{P}'^\sharp$ telle que $\mathcal{P}^\sharp \sqsubseteq^\sharp \mathcal{P}'^\sharp$.

skip

$$\frac{}{\rho \rightarrow^\flat_{\mathsf{skip}} \rho}$$

$$\begin{aligned}
\mathcal{P}^\sharp_{\mathsf{skip}}(\rho^\sharp) &= \alpha^{\mathit{Env}}(\{\rho \mid \exists \rho' \in \gamma^{\mathit{Env}}(\rho^\sharp), \rho' \rightarrow^\flat_{\mathsf{skip}} \rho\}) \\
&= \alpha^{\mathit{Env}}(\gamma^{\mathit{Env}}(\rho^\sharp)) \\
&\sqsubseteq \rho^\sharp
\end{aligned}$$

Affectation

$$\frac{\rho^\flat \vdash e \Downarrow_\epsilon \mathrm{v}}{\rho \rightarrow^\flat_{x:=e} \rho[x \mapsto v]}$$

$$\mathcal{P}^\sharp_{x:=e}(\rho^\sharp) = \alpha^{Env}(\{\rho[x \mapsto \mathrm{v}] \mid \rho \in \gamma^{Env}(\rho^\sharp), \rho \vdash e \Downarrow_\epsilon \mathrm{v}\})$$

Cas non relationnel

$$\begin{aligned}
\mathcal{P}^\sharp_{x:=e}(\rho^\sharp) &\sqsubseteq \alpha^\lambda(\alpha^{NR}(\{\rho[x \mapsto \mathrm{v}] \mid \rho \in \gamma^{Env}(\rho^\sharp), \mathrm{v} \in \gamma(E^\sharp(e, \rho^\sharp))\})) \\
&\sqsubseteq \alpha^\lambda(\lambda z.\{\rho[x \mapsto \mathrm{v}](z) \mid \rho \in \gamma^{Env}(\rho^\sharp), \mathrm{v} \in \gamma(E^\sharp(e, \rho^\sharp))\}) \\
&\sqsubseteq \begin{cases} \lambda z.\bot & \text{si } E^\sharp(e, \rho^\sharp) \sqsubseteq \text{Erreur} \\ \lambda z. \begin{cases} \rho^\sharp(z) & \text{si } z \neq x \\ E^\sharp(e, \rho^\sharp) & \text{si } z = x \end{cases} & \text{sinon} \end{cases}
\end{aligned}$$

Condition

$$\frac{\rho^\flat \vdash e \Downarrow_\epsilon \mathrm{v} \qquad v \neq 0}{\rho^\flat \rightarrow^\flat{}_{?e}\, \rho^\flat}$$

$$\mathcal{P}^\sharp_{?e}(\rho^\sharp) = \alpha^{Env}(\{\rho \mid \rho \in \gamma^{Env}(\rho^\sharp), \rho \vdash e \Downarrow_\epsilon \mathrm{v} \wedge \mathrm{v} \neq 0\})$$

Cas non relationnel

$$\begin{array}{rcl}
\mathcal{P}^\sharp_{?e}(\rho^\sharp) & = & \lambda z.\alpha\left(\begin{array}{l}\gamma(\rho^\sharp(z)) \cap \\ \{\rho(z) \mid \rho \in \gamma^{Env}(\rho^\sharp), \rho \vdash e \Downarrow_\epsilon \mathrm{v}, \mathrm{v} \neq 0\}\end{array}\right) \\
 & \sqsubseteq & \lambda z.\rho^\sharp(z) \sqcap \alpha(\{\rho(z) \mid \rho \in \gamma^{Env}(\rho^\sharp), \rho \vdash e \Downarrow_\epsilon \mathrm{v}, \mathrm{v} \neq 0\}) \\
 & \sqsubseteq & \begin{cases} \lambda z.\bot & \text{si } E^\sharp(e, \rho^\sharp) \sqsubseteq \alpha(\{0\}) \text{ ou } E^\sharp(e, \rho^\sharp) = \bot \\ \rho^\sharp & \text{sinon} \end{cases}
\end{array}$$

Transition plus précise

Remarque importante

Suivant le treillis et la forme de e, on peut raffiner la dernière inégalité.

On cherche à définir $\mathrm{test}^\sharp(e, z)$ telle que

$$\alpha(\{\rho(\mathsf{z}) \mid \rho \in \gamma^{Env}(\rho^\sharp), \rho \vdash e \Downarrow_\epsilon \mathrm{v} \wedge \mathrm{v} \neq 0\}) \sqsubseteq \mathrm{test}^\sharp(e, z)$$

$$\begin{aligned} \mathcal{P}^\sharp_{?e}(\rho^\sharp) & \sqsubseteq \lambda \mathsf{z}. \begin{array}{l} \alpha(\rho^\sharp(\mathsf{z})) \sqcap \\ \alpha\{\rho(\mathsf{z}) \mid \rho \in \gamma^{Env}(\rho^\sharp), \rho \vdash e \Downarrow_\epsilon \mathrm{v}, \mathrm{v} \neq 0\} \end{array} \\ & \sqsubseteq \lambda \mathsf{z}.\rho^\sharp(z) \sqcap \mathrm{test}^\sharp(e, z) \end{aligned}$$

Transition plus précise

Cas des signes

$$
\begin{aligned}
\mathrm{test}^\sharp(!\,x, x) &= \mathrm{Z}\\
\mathrm{test}^\sharp(x = e, x) &= E^\sharp(e, \rho^\sharp)\\
\mathrm{test}^\sharp(x <= e, x) &= \begin{cases} E^\sharp(e, \rho^\sharp) & \text{si SNEG} \sqsubseteq E^\sharp(e, \rho^\sharp) \sqsubseteq \text{NEG}\\ NEG & \text{si } E^\sharp(e, \rho^\sharp) = Z\\ \text{Int} & \text{sinon}\end{cases}
\end{aligned}
$$

Question

Que peut-on dire de $\mathrm{test}^\sharp(e <= y, y)$?

Erreur

$$\frac{\forall \mathrm{v}, \neg \rho^\flat \vdash e \Downarrow_\epsilon \mathrm{v}}{\rho^\flat \to^\flat{}_{Err(e)} \rho^\flat}$$

$$\begin{aligned}
\mathcal{P}^\sharp_{Err(e)}(\rho^\sharp) &= \alpha^{Env}(\{\rho \in \gamma^{Env}(\rho^\sharp) \mid \forall v \in \mathbb{Z}, \neg\rho \vdash e \Downarrow_\epsilon \mathrm{v}\}) \\
&\sqsubseteq \begin{cases} \alpha^{Env}(\gamma^{Env}(\rho^\sharp)) & \text{si } E^\sharp(e, \rho^\sharp) \in \{\bot, \top\} \\ \lambda z.\bot & \text{sinon} \end{cases} \\
&\sqsubseteq \begin{cases} \rho^\sharp & \text{si } E^\sharp(e, \rho^\sharp) \in \{\bot, \top\} \\ \lambda z.\bot & \text{sinon} \end{cases}
\end{aligned}$$

Treillis des intervalles

Domaine des intervalles

- $x \in [i_0, i_1]$
- domaine non relationnel
- conservation d'un intervalle regroupant toutes les valeurs possibles

Diagramme de Hasse

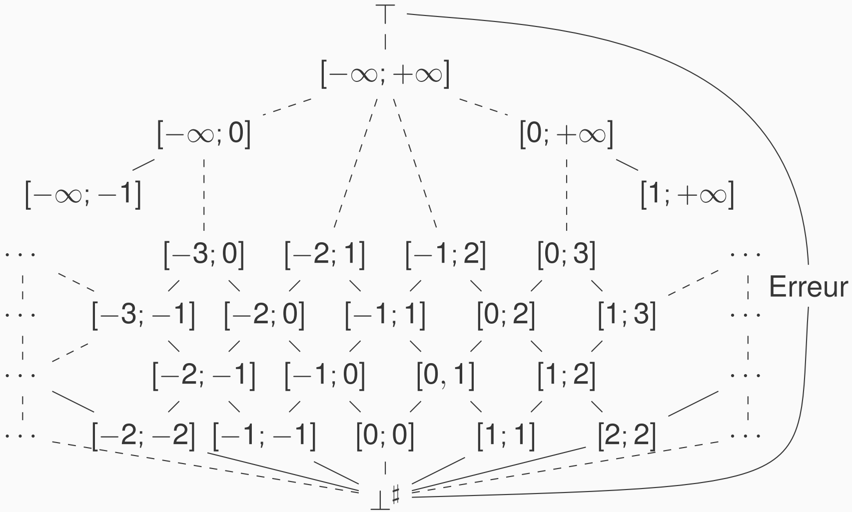
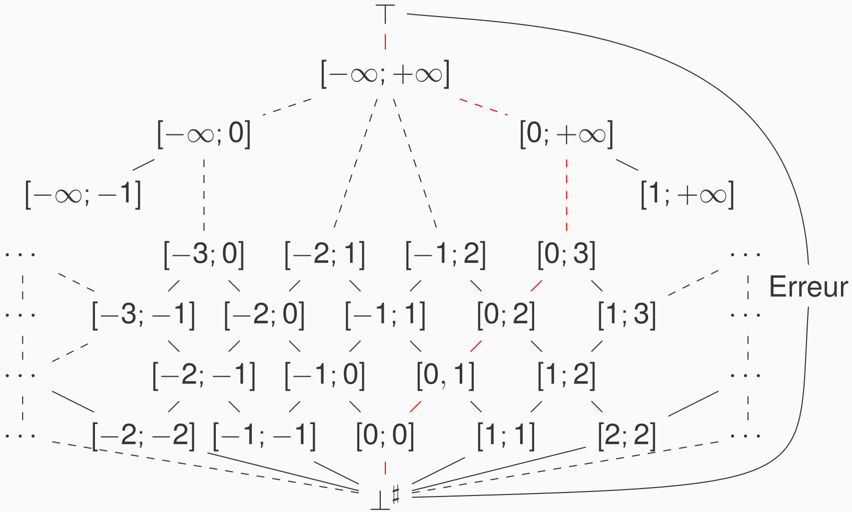


DIAGRAMME DE HASSE



Cadre générique

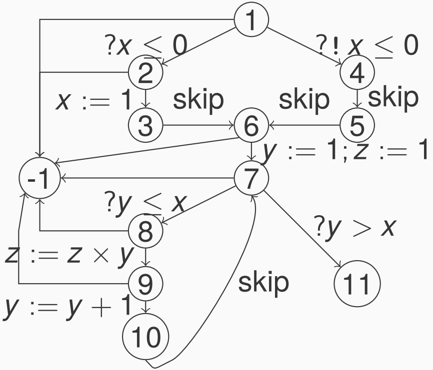
Même principe que pour le treillis des signes, mais on évalue les expressions dans le treillis des intervalles et non des signes.

Réduction d'état lors des tests

$$\begin{array}{l} \mathrm{test}^\sharp(!\,x, x) \\ \mathrm{test}^\sharp(x <= e, x) \\ \mathrm{test}^\sharp(x <= e, y) \\ \mathrm{test}^\sharp(e <= y, y) \end{array}$$

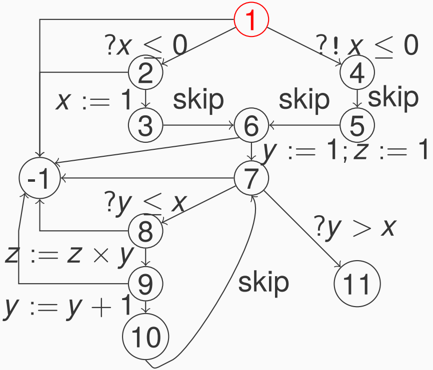
Opérateur d'élargissement

Position du problème



$$
\begin{aligned}
R(s_1) &= \bot^\sharp\\
R(s_2) &= \bot^\sharp\\
R(s_3) &= \bot^\sharp\\
R(s_4) &= \bot^\sharp\\
R(s_5) &= \bot^\sharp\\
R(s_6) &= \bot^\sharp\\
R(s_7) &= \bot^\sharp\\
R(s_8) &= \bot^\sharp\\
R(s_9) &= \bot^\sharp\\
R(s_{10}) &= \bot^\sharp\\
R(s_{11}) &= \bot^\sharp\\
R(s_{-1}) &= \bot^\sharp
\end{aligned}
$$

Position du problème



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$R(s_9) = \bot^\sharp$
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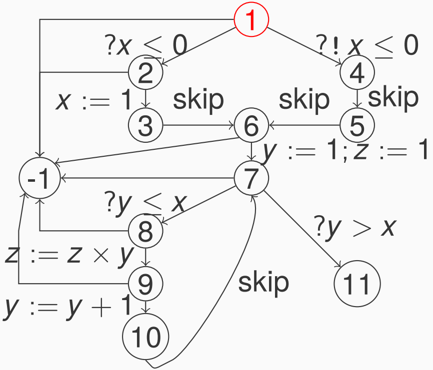
Position du problème



$$
\begin{aligned}
R(s_1) &= x \leftarrow [-\infty; +\infty] \\
R(s_2) &= \perp^\sharp \\
R(s_3) &= \perp^\sharp \\
R(s_4) &= \perp^\sharp \\
R(s_5) &= \perp^\sharp \\
R(s_6) &= \perp^\sharp \\
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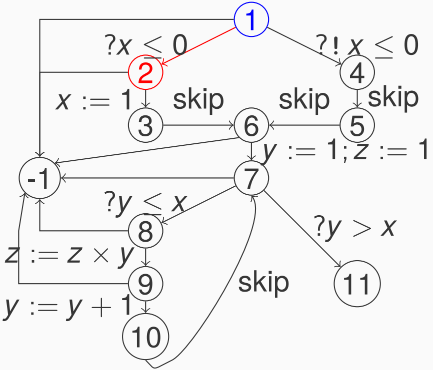
\end{aligned}
$$



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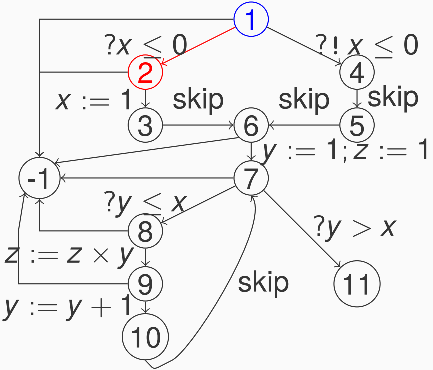

$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
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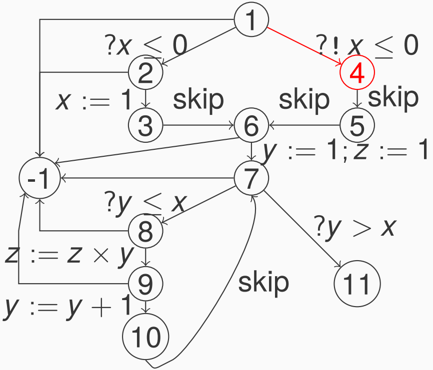
Position du problème



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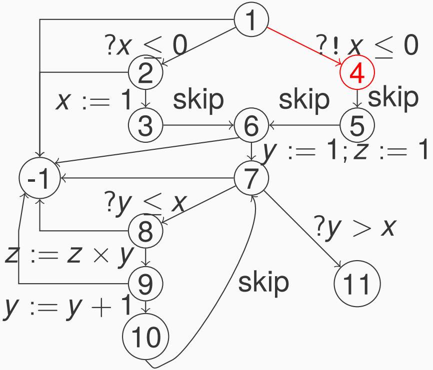
Position du problème



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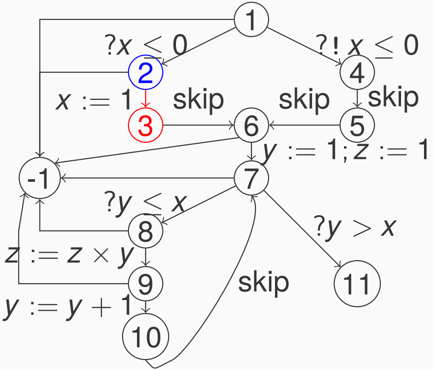

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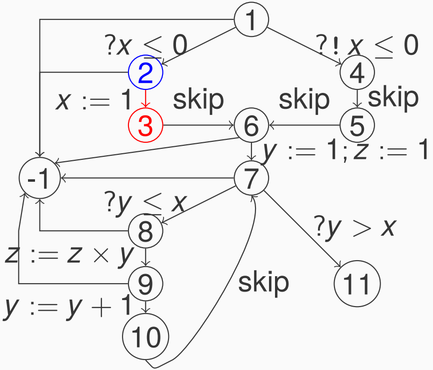
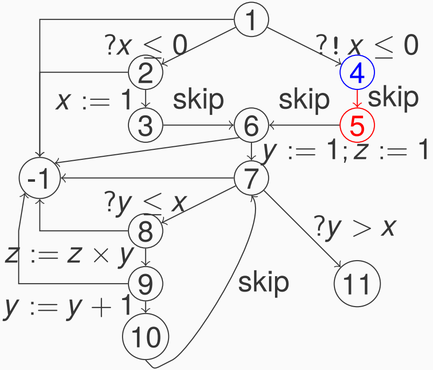
$$R(s_{-1}) = \perp^\sharp$$

Position du problème



$R(s_1)=x \leftarrow [-\infty;+\infty]$
$R(s_2)=x \leftarrow [-\infty;0]$
$R(s_3)=x \leftarrow [1;1]$
$R(s_4)=x \leftarrow [1;+\infty]$
$R(s_5)=\perp^\sharp$
$R(s_6)=\perp^\sharp$
$R(s_7)=\perp^\sharp$
$R(s_8)=\perp^\sharp$
$R(s_9)=\perp^\sharp$
$R(s_{10})=\perp^\sharp$
$R(s_{11})=\perp^\sharp$
$R(s_{-1})=\perp^\sharp$

Position du problème



$$
\begin{aligned}
R(s_1) &= x \leftarrow [-\infty; +\infty] \\
R(s_2) &= x \leftarrow [-\infty; 0] \\
R(s_3) &= x \leftarrow [1; 1] \\
R(s_4) &= x \leftarrow [1; +\infty] \\
R(s_5) &= \perp^\sharp \\
R(s_6) &= \perp^\sharp \\
R(s_7) &= \perp^\sharp \\
R(s_8) &= \perp^\sharp \\
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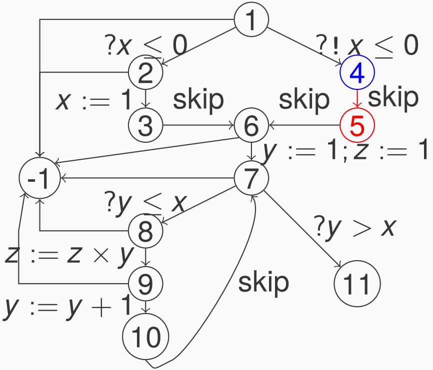
R(s_{11}) &= \perp^\sharp \\
R(s_{-1}) &= \perp^\sharp
\end{aligned}
$$

Position du problème



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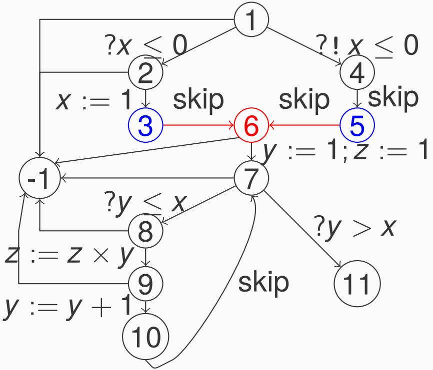
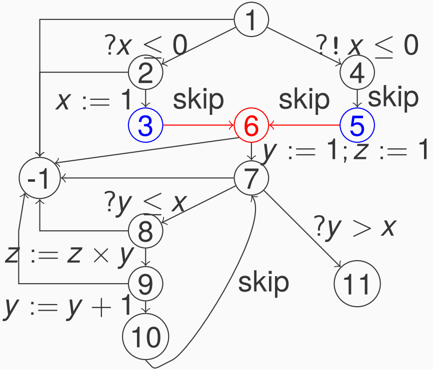
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = \perp^\sharp$
$R(s_7) = \perp^\sharp$
$R(s_8) = \perp^\sharp$
$R(s_9) = \perp^\sharp$
$R(s_{10}) = \perp^\sharp$
$R(s_{11}) = \perp^\sharp$
$R(s_{-1}) = \perp^\sharp$

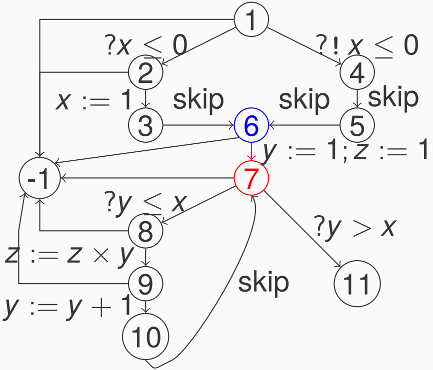


$$
\begin{aligned}
R(s_1) &= x \leftarrow [-\infty; +\infty] \\
R(s_2) &= x \leftarrow [-\infty; 0] \\
{\color{blue}R(s_3)} &= {\color{blue}x \leftarrow [1; 1]} \\
R(s_4) &= x \leftarrow [1; +\infty] \\
{\color{blue}R(s_5)} &= {\color{blue}x \leftarrow [1; +\infty]} \\
{\color{red}R(s_6)} &= \bot^\sharp \\
R(s_7) &= \bot^\sharp \\
R(s_8) &= \bot^\sharp \\
R(s_9) &= \bot^\sharp \\
R(s_{10}) &= \bot^\sharp \\
R(s_{11}) &= \bot^\sharp \\
R(s_{-1}) &= \bot^\sharp
\end{aligned}
$$



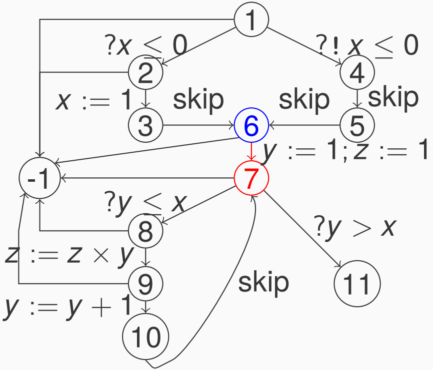
$$
\begin{aligned}
R(s_1) &= x \leftarrow [-\infty; +\infty] \\
R(s_2) &= x \leftarrow [-\infty; 0] \\
R(s_3) &= x \leftarrow [1; 1] \\
R(s_4) &= x \leftarrow [1; +\infty] \\
R(s_5) &= x \leftarrow [1; +\infty] \\
R(s_6) &= x \leftarrow [1; +\infty] \\
R(s_7) &= \perp^\sharp \\
R(s_8) &= \perp^\sharp \\
R(s_9) &= \perp^\sharp \\
R(s_{10}) &= \perp^\sharp \\
R(s_{11}) &= \perp^\sharp \\
R(s_{-1}) &= \perp^\sharp
\end{aligned}
$$

Position du problème



$$R(s_1)=x \leftarrow [-\infty; +\infty]$$
$$R(s_2)=x \leftarrow [-\infty; 0]$$
$$R(s_3)=x \leftarrow [1; 1]$$
$$R(s_4)=x \leftarrow [1; +\infty]$$
$$R(s_5)=x \leftarrow [1; +\infty]$$
$$R(s_6)=x \leftarrow [1; +\infty]$$
$$R(s_7)=\bot^\sharp$$
$$R(s_8)=\bot^\sharp$$
$$R(s_9)=\bot^\sharp$$
$$R(s_{10})=\bot^\sharp$$
$$R(s_{11})=\bot^\sharp$$
$$R(s_{-1})=\bot^\sharp$$

Position du problème



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

$R(s_4) = x \leftarrow [1; +\infty]$

$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_8) = \perp^\sharp$

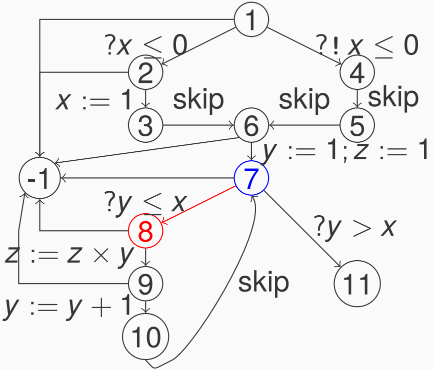
$R(s_9) = \perp^\sharp$

$R(s_{10}) = \perp^\sharp$

$R(s_{11}) = \perp^\sharp$

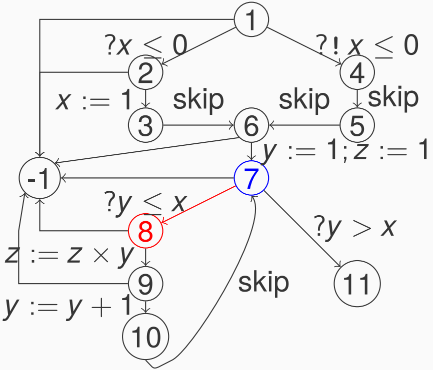
$R(s_{-1}) = \perp^\sharp$

Position du problème



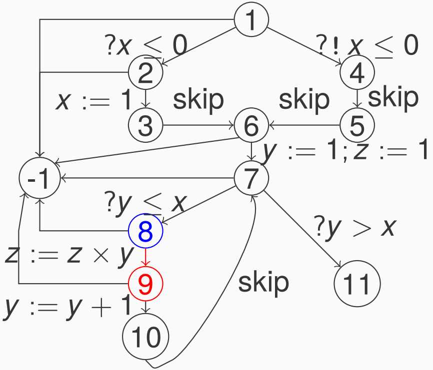
$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_8) = \perp^\sharp$
$R(s_9) = \perp^\sharp$
$R(s_{10}) = \perp^\sharp$
$R(s_{11}) = \perp^\sharp$
$R(s_{-1}) = \perp^\sharp$

Position du problème



$$R(s_1) = x \leftarrow [-\infty; +\infty]$$
$$R(s_2) = x \leftarrow [-\infty; 0]$$
$$R(s_3) = x \leftarrow [1; 1]$$
$$R(s_4) = x \leftarrow [1; +\infty]$$
$$R(s_5) = x \leftarrow [1; +\infty]$$
$$R(s_6) = x \leftarrow [1; +\infty]$$
$$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_9) = \bot^\sharp$$
$$R(s_{10}) = \bot^\sharp$$
$$R(s_{11}) = \bot^\sharp$$
$$R(s_{-1}) = \bot^\sharp$$

Position du problème



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

$R(s_4) = x \leftarrow [1; +\infty]$

$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

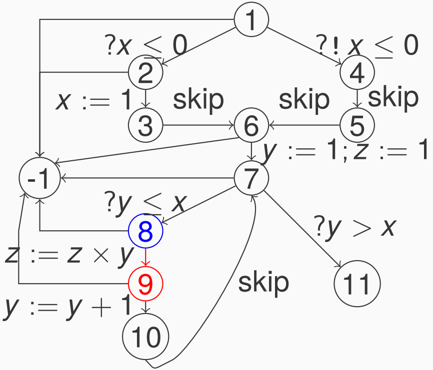
$R(s_9) = \bot^\sharp$

$R(s_{10}) = \bot^\sharp$

$R(s_{11}) = \bot^\sharp$

$R(s_{-1}) = \bot^\sharp$

Position du problème



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

$R(s_4) = x \leftarrow [1; +\infty]$

$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

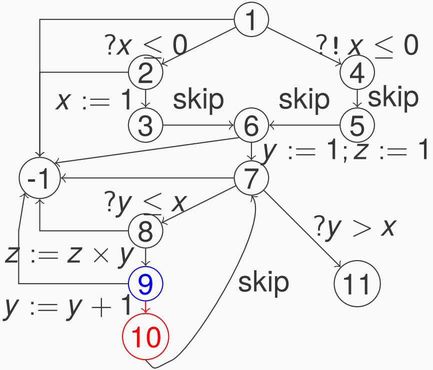
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_{10}) = \bot^\sharp$

$R(s_{11}) = \bot^\sharp$

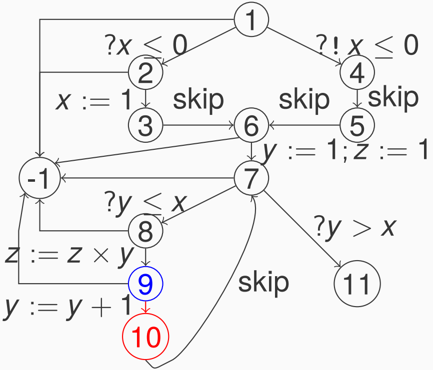
$R(s_{-1}) = \bot^\sharp$

Position du problème



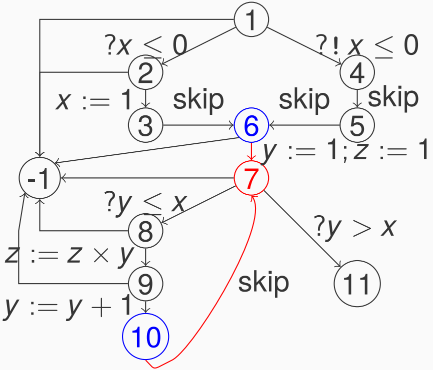
$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10}) = \bot^\sharp$
$R(s_{11}) = \bot^\sharp$
$R(s_{-1}) = \bot^\sharp$

Position du problème



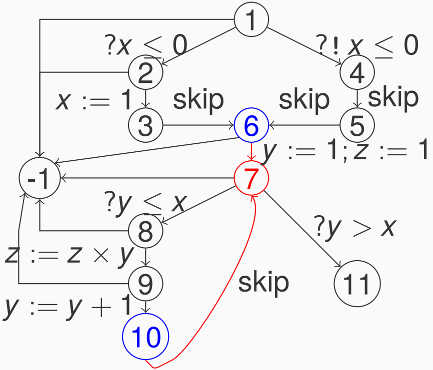
$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11}) = \perp^\sharp$
$R(s_{-1}) = \perp^\sharp$

Position du problème



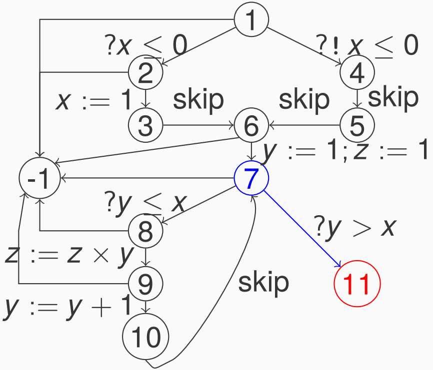
$$
\begin{aligned}
R(s_1) &= x \leftarrow [-\infty; +\infty] \\
R(s_2) &= x \leftarrow [-\infty; 0] \\
R(s_3) &= x \leftarrow [1; 1] \\
R(s_4) &= x \leftarrow [1; +\infty] \\
R(s_5) &= x \leftarrow [1; +\infty] \\
R(s_6) &= x \leftarrow [1; +\infty] \\
R(s_7) &= y \leftarrow [1; 1]; z \leftarrow [1; 1] \\
R(s_8) &= y \leftarrow [1; 1]; z \leftarrow [1; 1] \\
R(s_9) &= y \leftarrow [1; 1]; z \leftarrow [1; 1] \\
R(s_{10}) &= y \leftarrow [2; 2]; z \leftarrow [1; 1] \\
R(s_{11}) &= \perp^\sharp \\
R(s_{-1}) &= \perp^\sharp
\end{aligned}
$$

Position du problème



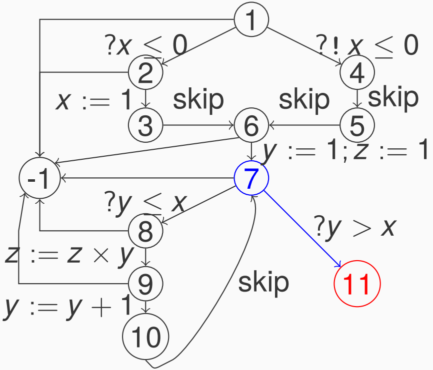
$$R(s_1) = x \leftarrow [-\infty; +\infty]$$
$$R(s_2) = x \leftarrow [-\infty; 0]$$
$$R(s_3) = x \leftarrow [1; 1]$$
$$R(s_4) = x \leftarrow [1; +\infty]$$
$$R(s_5) = x \leftarrow [1; +\infty]$$
$$R(s_6) = x \leftarrow [1; +\infty]$$
$$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$$
$$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$$
$$R(s_{11}) = \perp^\sharp$$
$$R(s_{-1}) = \perp^\sharp$$

Position du problème

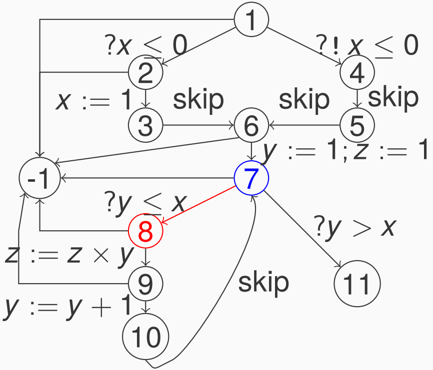


$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11}) = \bot^\sharp$
$R(s_{-1}) = \bot^\sharp$

Position du problème

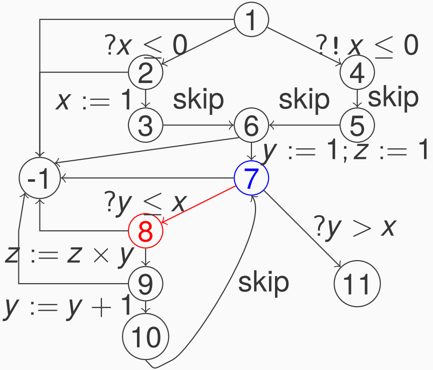


$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{-1}) = \perp^\sharp$



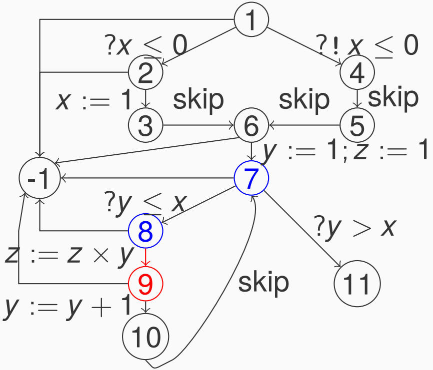
$$R(s_1) = x \leftarrow [-\infty; +\infty]$$
$$R(s_2) = x \leftarrow [-\infty; 0]$$
$$R(s_3) = x \leftarrow [1; 1]$$
$$R(s_4) = x \leftarrow [1; +\infty]$$
$$R(s_5) = x \leftarrow [1; +\infty]$$
$$R(s_6) = x \leftarrow [1; +\infty]$$
$$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$$
$$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$$
$$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$$
$$R(s_{-1}) = \perp^\sharp$$

Position du problème



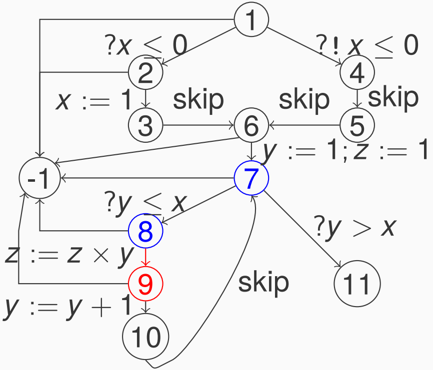
$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{-1}) = \perp^\sharp$

Position du problème



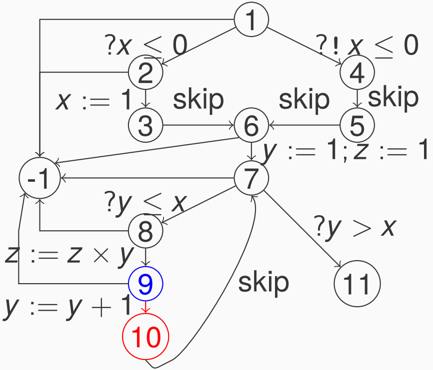
$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{-1}) = \perp^\sharp$

Position du problème



$R(s_1)$=$x \leftarrow [-\infty; +\infty]$
$R(s_2)$=$x \leftarrow [-\infty; 0]$
$R(s_3)$=$x \leftarrow [1; 1]$
$R(s_4)$=$x \leftarrow [1; +\infty]$
$R(s_5)$=$x \leftarrow [1; +\infty]$
$R(s_6)$=$x \leftarrow [1; +\infty]$
$R(s_7)$=$y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_8)$=$y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_9)$=$y \leftarrow [1; 2]; z \leftarrow [1; 2]$
$R(s_{10})$=$y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11})$=$y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{-1})$=$\perp^\sharp$

Position du problème



$R(s_1)$=$x \leftarrow [-\infty; +\infty]$

$R(s_2)$=$x \leftarrow [-\infty; 0]$

$R(s_3)$=$x \leftarrow [1; 1]$

$R(s_4)$=$x \leftarrow [1; +\infty]$

$R(s_5)$=$x \leftarrow [1; +\infty]$

$R(s_6)$=$x \leftarrow [1; +\infty]$

$R(s_7)$=$y \leftarrow [1; 2]; z \leftarrow [1; 1]$

$R(s_8)$=$y \leftarrow [1; 2]; z \leftarrow [1; 1]$

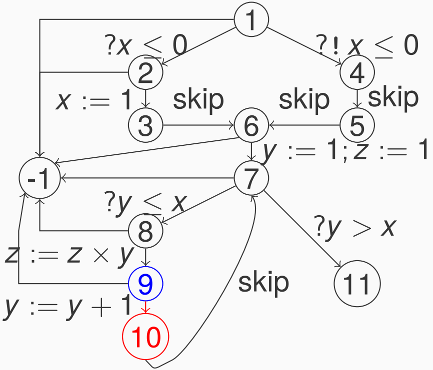
$R(s_9)$=$y \leftarrow [1; 2]; z \leftarrow [1; 2]$

$R(s_{10})$=$y \leftarrow [2; 2]; z \leftarrow [1; 1]$

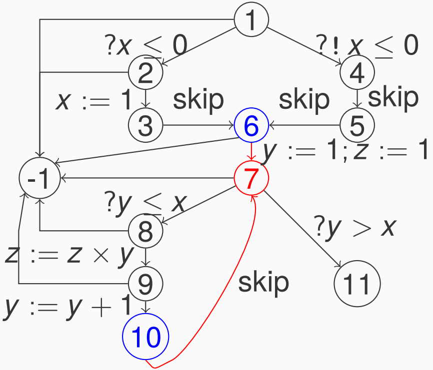
$R(s_{11})$=$y \leftarrow [2; 2]; z \leftarrow [1; 1]$

$R(s_{-1})$=$\perp^\sharp$

Position du problème

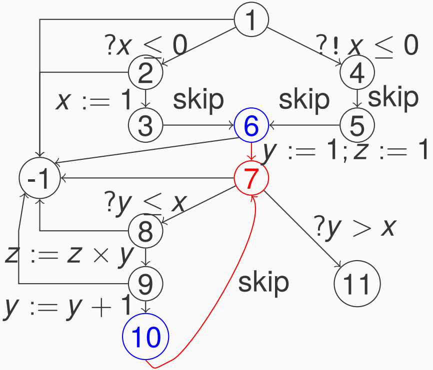


$$R(s_1) = x \leftarrow [-\infty; +\infty]$$
$$R(s_2) = x \leftarrow [-\infty; 0]$$
$$R(s_3) = x \leftarrow [1; 1]$$
$$R(s_4) = x \leftarrow [1; +\infty]$$
$$R(s_5) = x \leftarrow [1; +\infty]$$
$$R(s_6) = x \leftarrow [1; +\infty]$$
$$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$$
$$R(s_8) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$$
$$R(s_9) = y \leftarrow [1; 2]; z \leftarrow [1; 2]$$
$$R(s_{10}) = y \leftarrow [2; 3]; z \leftarrow [1; 2]$$
$$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$$
$$R(s_{-1}) = \bot^\sharp$$

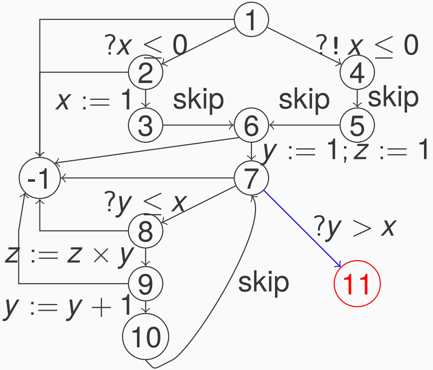


$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 2]; z \leftarrow [1; 2]$
$R(s_{10}) = y \leftarrow [2; 3]; z \leftarrow [1; 2]$
$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{-1}) = \perp^\sharp$

Position du problème

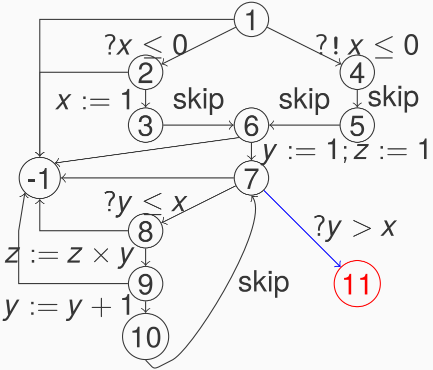


$$R(s_1) = x \leftarrow [-\infty; +\infty]$$
$$R(s_2) = x \leftarrow [-\infty; 0]$$
$$R(s_3) = x \leftarrow [1; 1]$$
$$R(s_4) = x \leftarrow [1; +\infty]$$
$$R(s_5) = x \leftarrow [1; +\infty]$$
$$R(s_6) = x \leftarrow [1; +\infty]$$
$$R(s_7) = y \leftarrow [1; 3]; z \leftarrow [1; 2]$$
$$R(s_8) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$$
$$R(s_9) = y \leftarrow [1; 2]; z \leftarrow [1; 2]$$
$$R(s_{10}) = y \leftarrow [2; 3]; z \leftarrow [1; 2]$$
$$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$$
$$R(s_{-1}) = \bot^\sharp$$



$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
$R(s_3) = x \leftarrow [1; 1]$
$R(s_4) = x \leftarrow [1; +\infty]$
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 3]; z \leftarrow [1; 2]$
$R(s_8) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 2]; z \leftarrow [1; 2]$
$R(s_{10}) = y \leftarrow [2; 3]; z \leftarrow [1; 2]$
$R(s_{11}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{-1}) = \perp^\sharp$

Position du problème



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

$R(s_4) = x \leftarrow [1; +\infty]$

$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

$R(s_7) = y \leftarrow [1; 3]; z \leftarrow [1; 2]$

$R(s_8) = y \leftarrow [1; 2]; z \leftarrow [1; 1]$

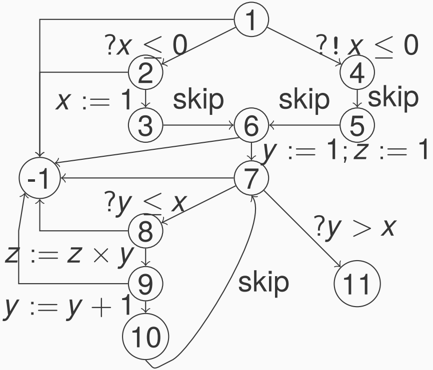
$R(s_9) = y \leftarrow [1; 2]; z \leftarrow [1; 2]$

$R(s_{10}) = y \leftarrow [2; 3]; z \leftarrow [1; 2]$

$R(s_{11}) = y \leftarrow [2, 3]; z \leftarrow [1; 2]$

$R(s_{-1}) = \perp^\sharp$

Position du problème



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R(s_8) &= y \leftarrow [1; +\infty]; z \leftarrow [1; +\infty]\\
R(s_9) &= y \leftarrow [1; +\infty]; z \leftarrow [1; +\infty]\\
R(s_{10}) &= y \leftarrow [2; +\infty]; z \leftarrow [1; +\infty]\\
R(s_{11}) &= y \leftarrow [2; +\infty]; z \leftarrow [1; +\infty]\\
R(s_{-1}) &= \perp^\sharp
\end{aligned}
$$

On veut obtenir une analyse qui termine toujours (et rapidement), même quand le treillis ne possède pas la propriété de chaîne ascendante.

Pour cela, on va utiliser une sur-approximation de la fonction de transition abstraite $F^\sharp$, construite de manière à garantir la terminaison.

Notre analyse consiste à calculer les itérations successives de $F^\sharp$ en partant de $\perp^\sharp$.
Avec $R^\sharp_n = {F^\sharp}^n(\perp^\sharp)$, on a $R^\sharp_n \sqsubseteq^\sharp R^\sharp_{n+1}$, et il vient :

$$R^\sharp_{n+1} \quad = \quad F^\sharp(R^\sharp_n)$$

Notre analyse consiste à calculer les itérations successives de $F^\sharp$ en partant de $\bot^\sharp$.
Avec $R_n^\sharp = {F^\sharp}^n(\bot^\sharp)$, on a $R_n^\sharp \sqsubseteq^\sharp R_{n+1}^\sharp$, et il vient :

$$
\begin{aligned}
R_{n+1}^\sharp &= F^\sharp(R_n^\sharp) \\
&= \begin{cases} R_n^\sharp & \text{si } F(R_n^\sharp) \sqsubseteq^\sharp R_n^\sharp \\ R_n^\sharp \sqcup^\sharp F^\sharp(R_n^\sharp) & \text{sinon} \end{cases}
\end{aligned}
$$

Notre analyse consiste à calculer les itérations successives de $F^\sharp$ en partant de $\bot^\sharp$.
Avec $R^\sharp_n = {F^\sharp}^n(\bot^\sharp)$, on a $R^\sharp_n \sqsubseteq^\sharp R^\sharp_{n+1}$, et il vient :

$$\begin{array}{rcl} R^\sharp_{n+1} & = & F^\sharp(R^\sharp_n) \\ & = & \begin{cases} R^\sharp_n & \text{si } F(R^\sharp_n) \sqsubseteq^\sharp R^\sharp_n \\ R^\sharp_n \sqcup^\sharp F^\sharp(R^\sharp_n) & \text{sinon} \end{cases} \end{array}$$

Retrouver la terminaison

Dans cette présentation, on dispose d'une nouvelle possibilité pour faire une approximation du point fixe : remplacer $\sqcup^\sharp$ par un opérateur donnant un autre majorant. Un tel opérateur s'appelle opérateur d'élargissement.

Définition

Un opérateur d'élargissement ∇ sur un treillis $(L, \sqsubseteq)$ est un opérateur binaire vérifiant les propriétés suivantes :

- $\forall x, y \in L, x \sqcup y \sqsubseteq x \nabla y$ *(majorant de ses arguments)*
- *Pour toute chaîne ascendante x_n, la chaîne $y_n = y_{n-1} \nabla x_n$ est stationnaire.*

Théorème

Soient un opérateur d'élargissement ∇ et une fonction monotone F, avec :

$$\begin{array}{rcl} R_0 & = & \bot^\sharp \\ R_{n+1} & = & \begin{cases} R_n & \text{si } F(R_n) \sqsubseteq R_n \\ R_n \nabla F(R_n) & \text{sinon} \end{cases} \end{array}$$

R_n est stationnaire à partir d'un certain rang N. De plus $\mathrm{lfp}(F) \sqsubseteq R_N$

Pour le treillis des intervalles, un exemple simple d'opérateur d'élargissement est le suivant :

$$\begin{array}{rcl}
\bot^\sharp \nabla v^\sharp & = & v^\sharp \\
v^\sharp \nabla \bot^\sharp & = & v^\sharp \\
\top^\sharp \nabla v^\sharp & = & \top^\sharp \\
v^\sharp \nabla \top^\sharp & = & \top^\sharp \\
[i^1_{min}; i^1_{max}] \nabla [i^2_{min}; i^2_{max}] & = & [i_{min}; i_{max}] \quad \text{avec} \left\{ \begin{array}{rl} i_{min} = i^1_{min} & \text{si } i^1_{min} \leq i^2_{min} \\ = -\infty & \text{sinon} \\ i_{max} = i^1_{max} & \text{si } i^2_{max} \leq i^1_{max} \\ = +\infty & \text{sinon} \end{array} \right.
\end{array}$$

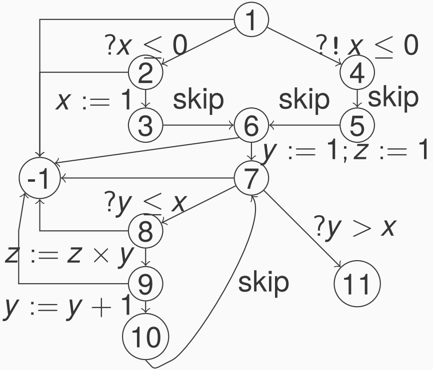
On étend cet opérateur au environnements abstraits :

$$\rho^\sharp_1 \nabla' \rho^\sharp_2 = \lambda x . \rho^\sharp_1(x) \nabla \rho^\sharp_2(x)$$

Calcul de point fixe avec élargissement

procedure ChaoticIteration(f_e, i)
    $\sigma[\mathcal{I}] \leftarrow i$; $WorkList \leftarrow \{(\mathcal{I}, s) | s \in \mathrm{Succ}(\mathcal{I})\}$
    while $WorkList \neq \emptyset$ **do**
        $(s, s') \leftarrow Choose(WorkList)$; $WorkList \leftarrow WorkList \setminus \{(s, s')\}$
        $v \leftarrow f_{\epsilon(s,s')}(\sigma[s])\}$
        if $\neg v \sqsubseteq \sigma[s']$ **then**
            if (s, s') est une arête de retour **then**
                $\sigma[s'] \leftarrow \sigma[s'] \nabla v$
            else
                $\sigma[s'] \leftarrow \sigma[s'] \sqcup v$
            end if
            $WorkList \leftarrow WorkList \cup \{(s', s'') | s'' \in \mathrm{Succ}(s')\}$
        end if
    end while
end procedure

Exemple d'élargissement



$$
\begin{aligned}
R(s_1) &= \bot^\sharp \\
R(s_2) &= \bot^\sharp \\
R(s_3) &= \bot^\sharp \\
R(s_4) &= \bot^\sharp \\
R(s_5) &= \bot^\sharp \\
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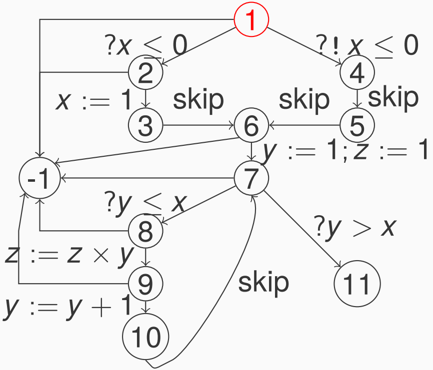
R(s_7) &= \bot^\sharp \\
R(s_8) &= \bot^\sharp \\
R(s_9) &= \bot^\sharp \\
R(s_{10}) &= \bot^\sharp \\
R(s_{11}) &= \bot^\sharp \\
R(s_{-1}) &= \bot^\sharp
\end{aligned}
$$

Exemple d'élargissement



$R(s_1) = \perp^\sharp$
$R(s_2) = \perp^\sharp$
$R(s_3) = \perp^\sharp$
$R(s_4) = \perp^\sharp$
$R(s_5) = \perp^\sharp$
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$R(s_8) = \perp^\sharp$
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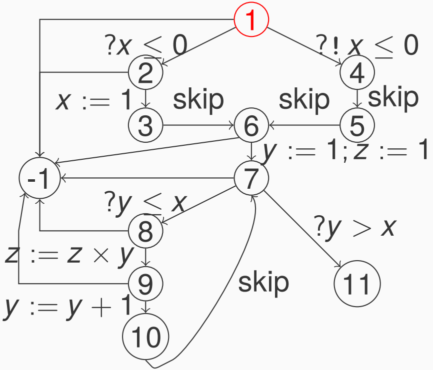
Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = \bot^\sharp$
$R(s_3) = \bot^\sharp$
$R(s_4) = \bot^\sharp$
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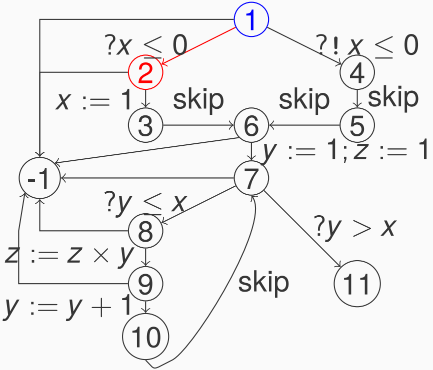
Exemple d'élargissement



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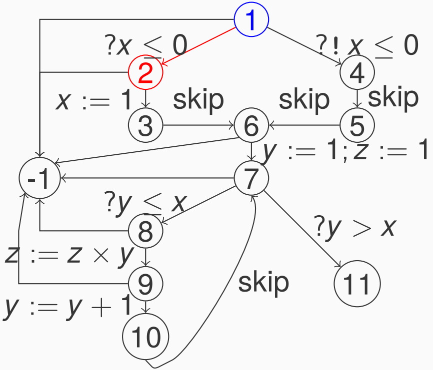
Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = \bot^\sharp$

$R(s_4) = \bot^\sharp$

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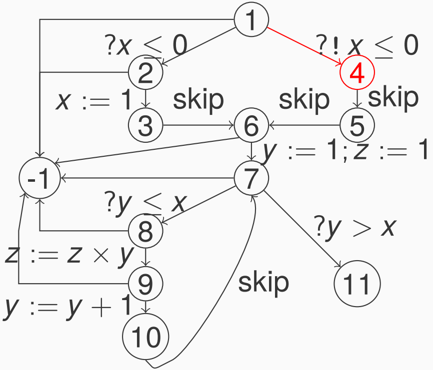
Exemple d'élargissement



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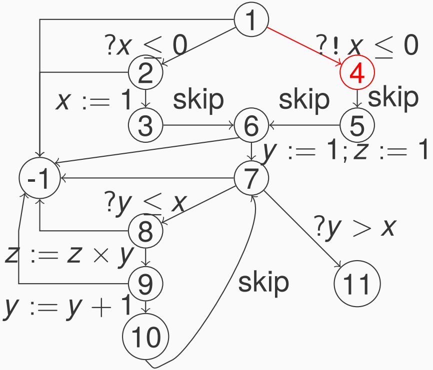
Exemple d'élargissement



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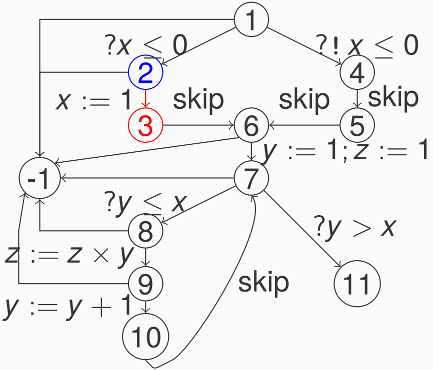
Exemple d'élargissement



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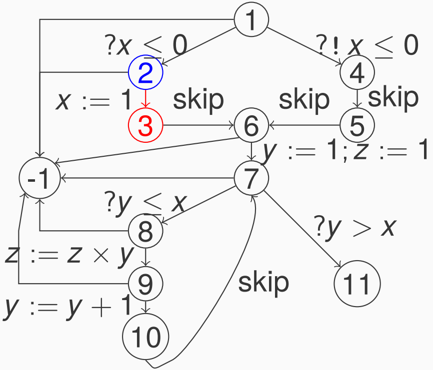
Exemple d'élargissement



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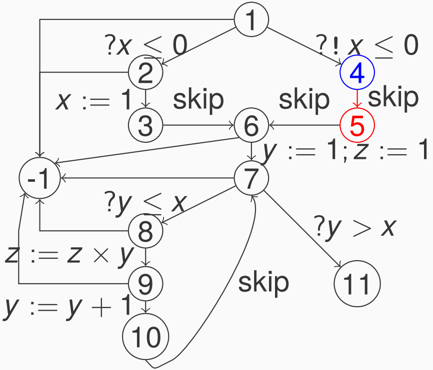
Exemple d'élargissement



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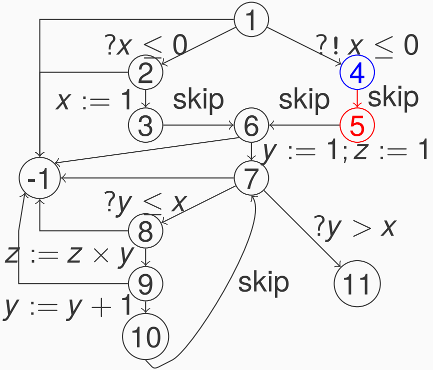
Exemple d'élargissement



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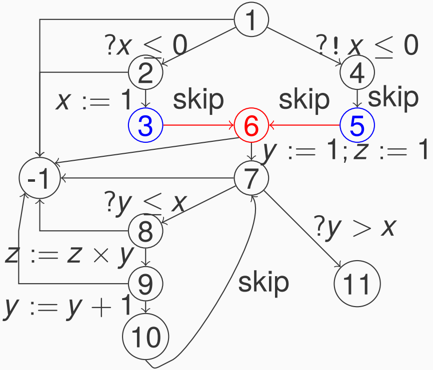
Exemple d'élargissement



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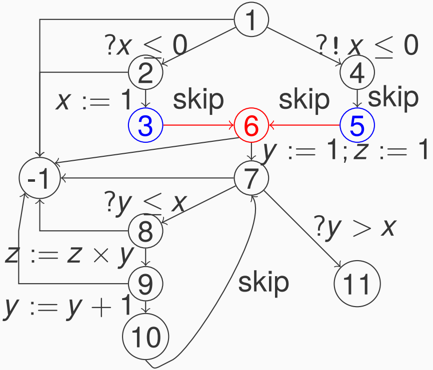
\end{aligned}
$$

Exemple d'élargissement



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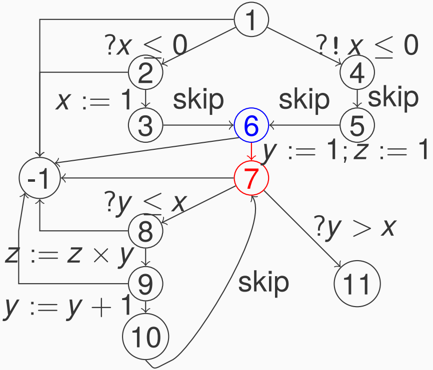
Exemple d'élargissement



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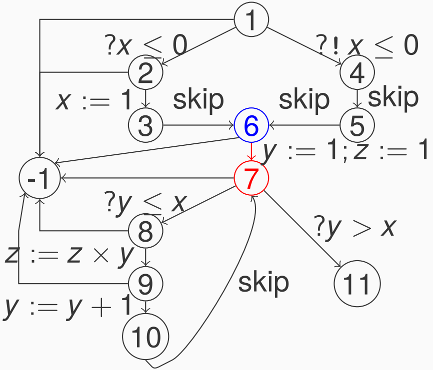
R(s_{11}) &= \perp^\sharp \\
R(s_{-1}) &= \perp^\sharp
\end{aligned}
$$

EXEMPLE D'ÉLARGISSEMENT



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$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_8) = \perp^\sharp$
$R(s_9) = \perp^\sharp$
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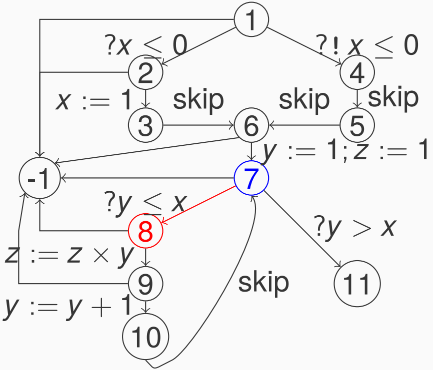
$R(s_{11}) = \perp^\sharp$
$R(s_{-1}) = \perp^\sharp$

Exemple d'élargissement



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$$R(s_6) = x \leftarrow [1; +\infty]$$
$$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_8) = \bot^\sharp$$
$$R(s_9) = \bot^\sharp$$
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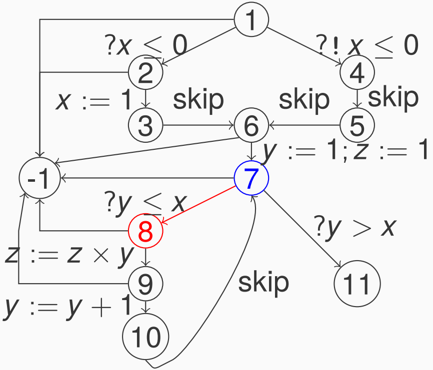
Exemple d'élargissement



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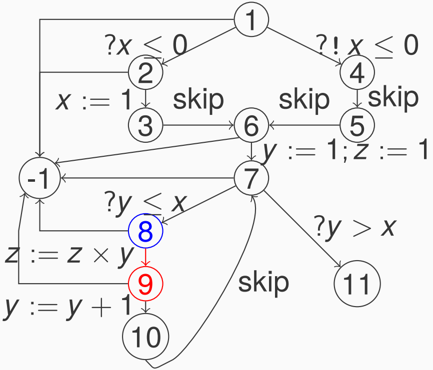
Exemple d'élargissement



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$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

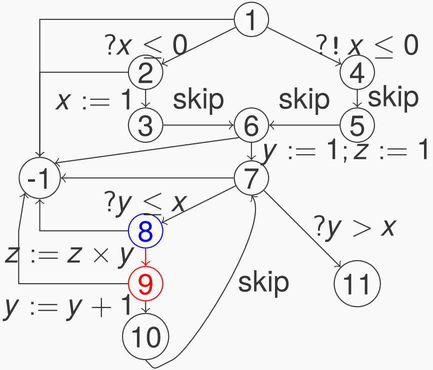
$R(s_9) = \bot^\sharp$

$R(s_{10}) = \bot^\sharp$

$R(s_{11}) = \bot^\sharp$

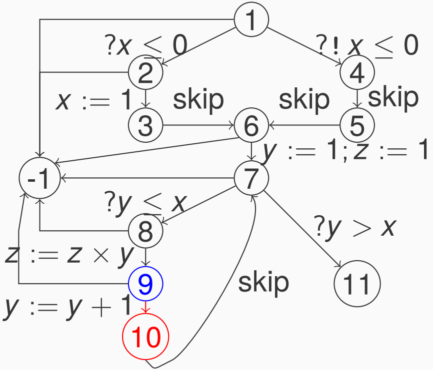
$R(s_{-1}) = \bot^\sharp$

Exemple d'élargissement



$$R(s_1) = x \leftarrow [-\infty; +\infty]$$
$$R(s_2) = x \leftarrow [-\infty; 0]$$
$$R(s_3) = x \leftarrow [1; 1]$$
$$R(s_4) = x \leftarrow [1; +\infty]$$
$$R(s_5) = x \leftarrow [1; +\infty]$$
$$R(s_6) = x \leftarrow [1; +\infty]$$
$$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$$
$$R(s_{10}) = \perp^\sharp$$
$$R(s_{11}) = \perp^\sharp$$
$$R(s_{-1}) = \perp^\sharp$$

Exemple d'élargissement



$R(s_1)$=$x \leftarrow [-\infty; +\infty]$
$R(s_2)$=$x \leftarrow [-\infty; 0]$
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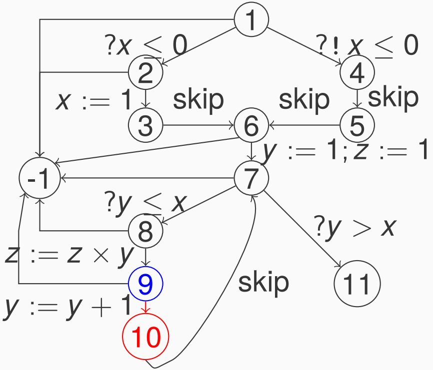
$R(s_4)$=$x \leftarrow [1; +\infty]$
$R(s_5)$=$x \leftarrow [1; +\infty]$
$R(s_6)$=$x \leftarrow [1; +\infty]$
$R(s_7)$=$y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_8)$=$y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_9)$=$y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10})$=$\perp^\sharp$
$R(s_{11})$=$\perp^\sharp$
$R(s_{-1})$=$\perp^\sharp$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

$R(s_4) = x \leftarrow [1; +\infty]$

$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

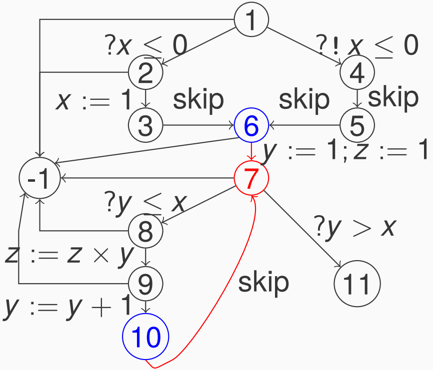
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$

$R(s_{11}) = \perp^\sharp$

$R(s_{-1}) = \perp^\sharp$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
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$R(s_5) = x \leftarrow [1; +\infty]$
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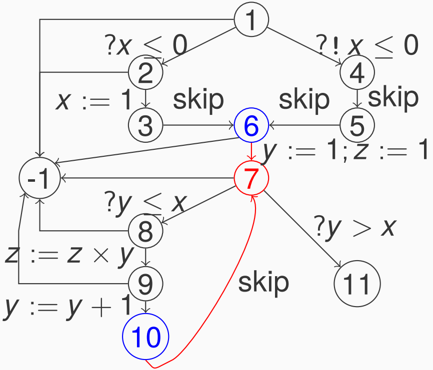
$R(s_7) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11}) = \perp^\sharp$
$R(s_{-1}) = \perp^\sharp$

Exemple d'élargissement



$$
\begin{array}{l}
R(s_1) = x \leftarrow [-\infty; +\infty] \\
R(s_2) = x \leftarrow [-\infty; 0] \\
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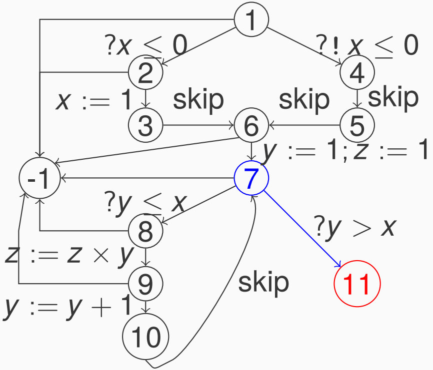
R(s_5) = x \leftarrow [1; +\infty] \\
R(s_6) = x \leftarrow [1; +\infty] \\
R(s_7) = y \leftarrow [1; +\infty]; z \leftarrow [1; 1] \\
R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1] \\
R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1] \\
R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1] \\
R(s_{11}) = \perp^\sharp \\
R(s_{-1}) = \perp^\sharp
\end{array}
$$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

$R(s_4) = x \leftarrow [1; +\infty]$

$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

$R(s_7) = y \leftarrow [1; +\infty]; z \leftarrow [1; 1]$

$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

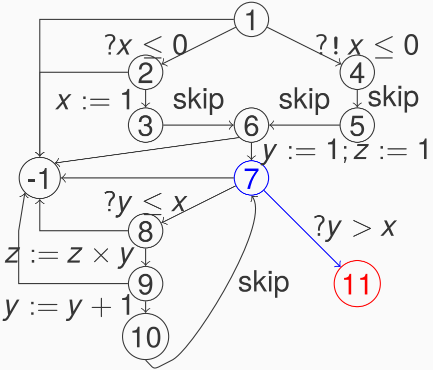
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$

$R(s_{11}) = \bot^\sharp$

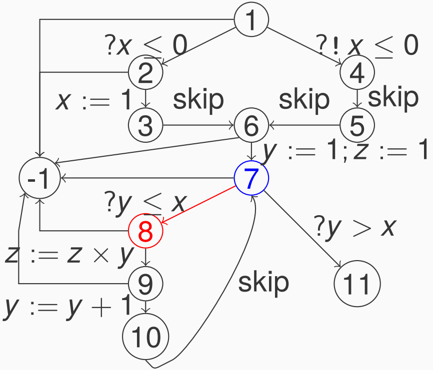
$R(s_{-1}) = \bot^\sharp$

EXEMPLE D'ÉLARGISSEMENT



$R(s_1)=x \leftarrow [-\infty; +\infty]$
$R(s_2)=x \leftarrow [-\infty; 0]$
$R(s_3)=x \leftarrow [1; 1]$
$R(s_4)=x \leftarrow [1; +\infty]$
$R(s_5)=x \leftarrow [1; +\infty]$
$R(s_6)=x \leftarrow [1; +\infty]$
$R(s_7)=y \leftarrow [1; +\infty]; z \leftarrow [1; 1]$
$R(s_8)=y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_9)=y \leftarrow [1; 1]; z \leftarrow [1; 1]$
$R(s_{10})=y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11})=y \leftarrow [2; +\infty]; z \leftarrow [1; 1]$
$R(s_{-1})=\perp^\sharp$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

$R(s_4) = x \leftarrow [1; +\infty]$

$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

$R(s_7) = y \leftarrow [1; +\infty]; z \leftarrow [1; 1]$

$R(s_8) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

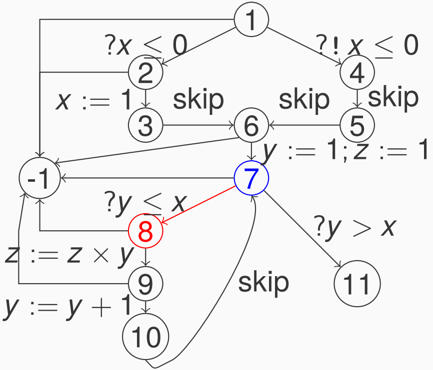
$R(s_9) = y \leftarrow [1; 1]; z \leftarrow [1; 1]$

$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$

$R(s_{11}) = y \leftarrow [2; +\infty]; z \leftarrow [1; 1]$

$R(s_{-1}) = \perp^\sharp$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
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$R(s_4) = x \leftarrow [1; +\infty]$
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$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; +\infty]; z \leftarrow [1; 1]$
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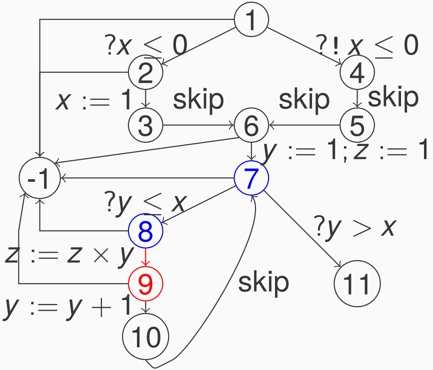
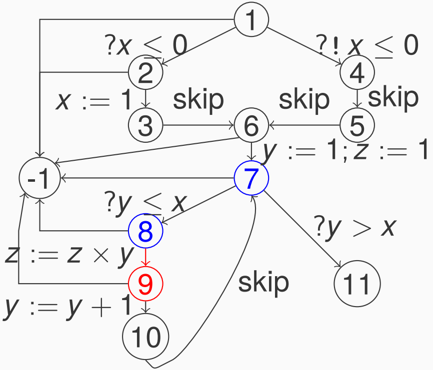
$R(s_{11}) = y \leftarrow [2; +\infty]; z \leftarrow [1; 1]$
$R(s_{-1}) = \perp^\sharp$

EXEMPLE D'ÉLARGISSEMENT



$$
\begin{aligned}
R(s_1) &= x \leftarrow [-\infty; +\infty] \\
R(s_2) &= x \leftarrow [-\infty; 0] \\
R(s_3) &= x \leftarrow [1; 1] \\
R(s_4) &= x \leftarrow [1; +\infty] \\
R(s_5) &= x \leftarrow [1; +\infty] \\
R(s_6) &= x \leftarrow [1; +\infty] \\
R(s_7) &= y \leftarrow [1; +\infty]; z \leftarrow [1; 1] \\
R(s_8) &= y \leftarrow [1; +\infty]; z \leftarrow [1; 1] \\
R(s_9) &= y \leftarrow [1; 1]; z \leftarrow [1; 1] \\
R(s_{10}) &= y \leftarrow [2; 2]; z \leftarrow [1; 1] \\
R(s_{11}) &= y \leftarrow [2; +\infty]; z \leftarrow [1; 1] \\
R(s_{-1}) &= \bot^\sharp
\end{aligned}
$$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$
$R(s_2) = x \leftarrow [-\infty; 0]$
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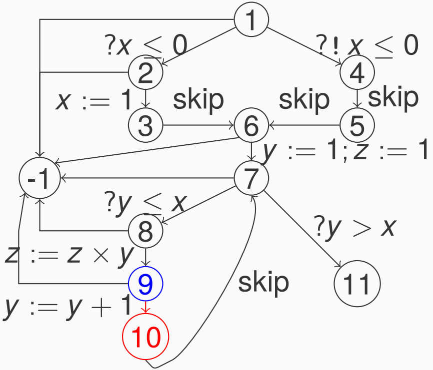
$R(s_5) = x \leftarrow [1; +\infty]$
$R(s_6) = x \leftarrow [1; +\infty]$
$R(s_7) = y \leftarrow [1; +\infty]; z \leftarrow [1; 1]$
$R(s_8) = y \leftarrow [1; +\infty]; z \leftarrow [1; 1]$
$R(s_9) = y \leftarrow [1; +\infty]; z \leftarrow [1; +\infty]$
$R(s_{10}) = y \leftarrow [2; 2]; z \leftarrow [1; 1]$
$R(s_{11}) = y \leftarrow [2; +\infty]; z \leftarrow [1; 1]$
$R(s_{-1}) = \perp^\sharp$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

$R(s_2) = x \leftarrow [-\infty; 0]$

$R(s_3) = x \leftarrow [1; 1]$

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$R(s_5) = x \leftarrow [1; +\infty]$

$R(s_6) = x \leftarrow [1; +\infty]$

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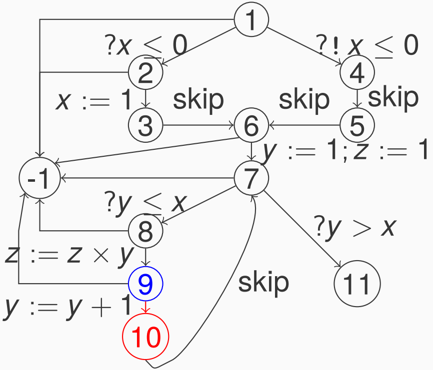
$R(s_{-1}) = \perp^\sharp$

EXEMPLE D'ÉLARGISSEMENT



$R(s_1) = x \leftarrow [-\infty; +\infty]$
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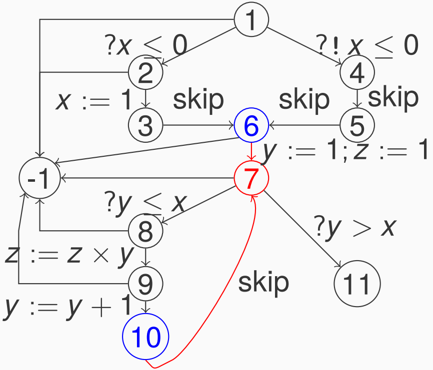
$R(s_{10}) = y \leftarrow [2; +\infty]; z \leftarrow [1; +\infty]$
$R(s_{11}) = y \leftarrow [2; +\infty]; z \leftarrow [1; 1]$
$R(s_{-1}) = \bot^\sharp$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

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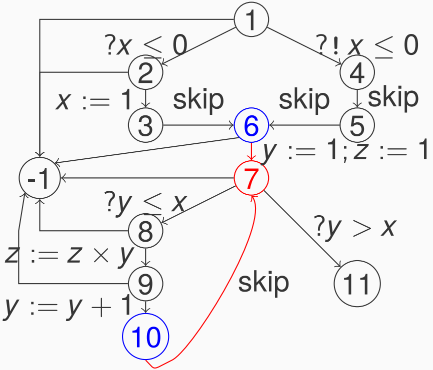
Exemple d'élargissement



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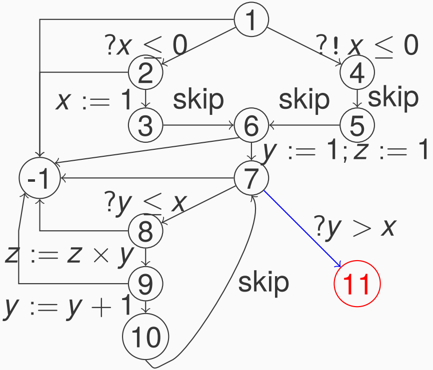
Exemple d'élargissement



$R(s_1)$=$x \leftarrow [-\infty; +\infty]$
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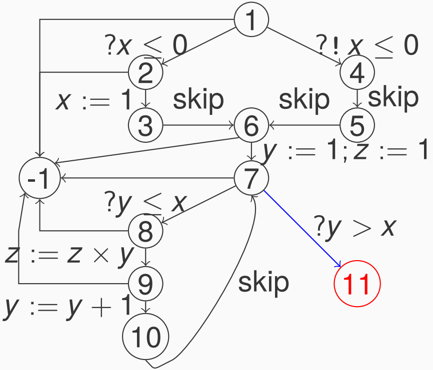
$R(s_{-1})$=$\perp^\sharp$

Exemple d'élargissement



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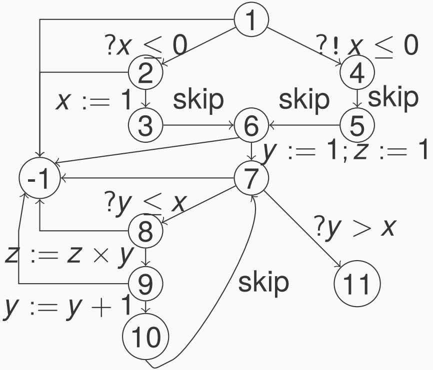
$$R(s_{-1})=\bot^\sharp$$

EXEMPLE D'ÉLARGISSEMENT



$R(s_1) = x \leftarrow [-\infty; +\infty]$

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$R(s_{10}) = y \leftarrow [2; +\infty]; z \leftarrow [1; +\infty]$

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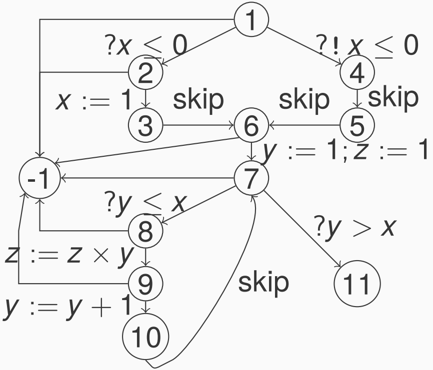
$R(s_{-1}) = \perp^\sharp$

Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

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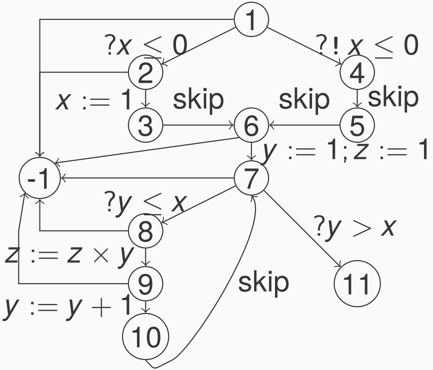
Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$
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$R(s_{10}) = y \leftarrow [2; +\infty]; z \leftarrow [1; +\infty]$
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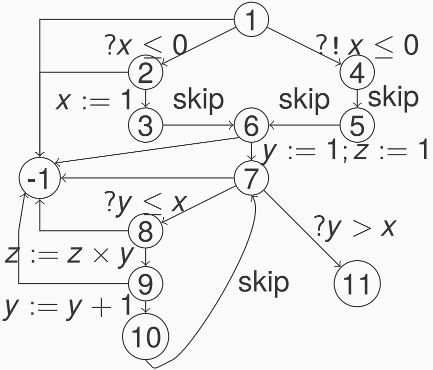
Exemple d'élargissement



$R(s_1) = x \leftarrow [-\infty; +\infty]$

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